Courses of the PhD Program in Mathematics, a.y. 2013/14

1. **Algebra and mathematical logic**

- Professors: Michele Elia (Politecnico di Torino), Teo Ferdinando Mora (Univ. di Genova) and Massimiliano Sala
- Title: **Complexity, Coding and Cryptography**
- Program: We will discuss some algorithms to solve algebraic problems useful in coding theory and cryptography, with emphasis on their complexity. M. Elia will focus on the univariate case, while T. Mora will focus on the multivariate case. M. Sala will show some links between the computational approach and the applications..
- Period: March 31 - April 11, 2014, intensive course.

2. **Geometry**

- Professor: R. Pignatelli
- Title: **Complex Algebraic Geometry**
- The course is attended also from Master degree students of Mathematics. The following is a reduced program for the PhD Program.
- Program: The course will give an introduction to the theory of the algebraic varieties, with emphasys on the dimension 2 case.
- Period: February 17, 2013–April 30, 2014, about 4 hours per week.

3. **Mathematical analysis**

- Professor: A. Visintin
- Title: **Partial Differential Equations**
- The course is attended also from Master degree students of Mathematics. The following is a reduced program for the PhD Program.
- Outline: The modern theory of PDEs is based upon several analytical tools. In particular it has been and is still developed in function spaces, and thus rests upon functional analysis. This course will introduce some basic tenets of that theory, essentially the Schwartz distributions and Sobolev spaces. The final goal will be the weak formulation of boundary- and initial-value problems for second-order PDEs.
- Program:
  Review of the elementary theory of PDE:
  Linear first order equations and basic linear second order equations:Laplace, heat and wave equations.
  Basics of boundary- and initial-value problems for linear PDEs:
  Resolution via variable separation. Characteristics. Classification of PDEs.
  Main contents:
  Schwartz distributions.
  Convolution, Fundamental solutions, Green functions.
  Fourier transform in the Schwartz space and application to PDEs with constant coefficients.
  Conservation laws.
- Period: February 17, 2014–April 30, 2014, about 5 hours per week.

For the PhD students in the research area of biomathematics/computational biology the following course is recommended:

- Professor: O. Diekmann (Utrecht University)
- Title: **Structured Population Models, Delay Equations and Infinite-Dimensional Dynamical Systems**

- Program: The aim of this course is to introduce three related topics :

• physiologically structured population models, i.e., models in which individuals are distinguished from one another according to age, size, ...

• delay equations, by which we mean RE (renewal equations, i.e., Volterra integral equations), DDE (delay differential equations) and mixed systems of these

• dynamical systems on Banach spaces (in the linear case one usually speaks about semigroups of bounded linear operators and their infinitesimal generators).

By way of examples attention is paid to the biological motivation and interpretation as well as to the derivation of delay equations from modelling assumptions. The relevant theory for delay equations will be sketched against the general background of dynamical systems, with special attention for stability and bifurcation of steady states. In this context characteristic equations play a major role and a few examples of such characteristic equations will be discussed in detail.

- Period: March 19, 2014–April 19,2014.

4. **Probability, statistics and mathematical methods for economy**

- Professors: S. Bonaccorsi, D. Mugnolo (University of Ulm)
- Title: **Evolution equations on networks**
- Program: 1. Operators on networks (Difference operators on Graphs, Differential operators on metric graphs, Hybrid operators on metric graphs) 2. Function spaces on networks (the discrete and the continuous settings) 3. Evolution equations associated with self-adjoint operators (Operator semigroups, sesquilinear forms and analytic semigroups, diffusion equations, Schroedinger equations, wave equations, beam equations on network, quantum graphs) 4. Stochastic evolution equations (stochastic processes, Brownian motion, diffusion equations for the Brownian motion, diffusion equations perturbed by Brownian motion)
- Period: February 10-21, 2014 or June 2-13,2013, to be confirmed, intensive course.

5. **Mathematical physics**

- Professor: V. Moretti
- Title: **Mathematical physics**
- The course is attended also from Master degree students of Mathematics and Physics.
- Program: 1) Elements of differential geometry (including tensor analysis) applied to Relativistic Theories. (2) Elements of Hilbert space functional analysis applied to quantum theories..
- Period: February 17, 2014–April 30, 2014, about 4 hours per week.

6. **Numerical analysis**

- Professors: A. Alonso Rodriguez, A. Valli
- Title: **Mathematical Methods for Engineering- Part II**
- Summary: The second part of the winter school on numerical methods consists of a two- week intensive programme of 34 hours of theoretical lectures and 16 hours of computer laboratory exercises. The focus is on the presentation of some mathematical results that are at the basis of numerical approximation for partial differential equations, and on the analysis of some of these numerical procedures (boundary element, finite element and spectral methods). Theoretical results are described and proved (though not in complete detail), with the aim of placing the numerical methods on a solid ground and permitting their stability and convergence analysis. The algebraic structure of the discrete problems is also presented and analysed. The tutorials are devoted to the effective im- plementation in laboratory of these approximation schemes, using MATLAB and the software Freefem. The course is mainly directed to PhD students and post-doctoral researchers in applied mathematics, engineering, and other scientific disciplines.
- Program:

• Theory: -) Partial differential equations (elliptic equations, parabolic equations, hyperbolic equations, boundary value problems). -)Separation of variables (solution of heat and wave equations by means of Fourier expansion, orthonormal bases, Sturm-Liouville problems, Bessel functions, Legendre and Chebyshev polynomials). -)Fundamental solutions and Green functions for elliptic equations (Dirac delta ”function”, distributions, fundamental solutions, Green functions, integral representation formula in terms of the Green function). -)Integral equations and the boundary element method for elliptic problems (Green formulae, interior and boundary integral representation formulae in terms of the fundamental solution, integral equation on the boundary, collocation and Galerkin formulations of the boundary element method, algebraic structure of the approximating problems). -)Weak formulation and the finite element method for elliptic problems (minimization problems, Euler equation of a functional, weak formulation, Lax-Milgram lemma, existence and uniqueness of the solution, Galerkin approximation, finite element methods and spectral methods, family of triangulations and basis functions, Ca lemma and error esti- mates, mixed formulation and Stokes problem, mixed finite element me- thods, Ladyzhenskaya-Babuska-Brezzi condition and error estimates, compatible choices of finite elements, algebraic structure of the discrete problems, other applications).

• Tutorials: -)The boundary element method: remarks on programming. -)The finite element method, 1 (classical formulations) & programming -)The finite element method, 2 (mixed formulations) & programming -)FreeFEM: an example of finite element software.

- Period: February 24, 2014- March 7, 2014, intensive course.

7. **Operational research**

- Professor: R. Rizzi
- Title: **Mathematical Programming**
- Outline: The course offers an introduction to Linear Programming (LP) and Combinatorial Optimization (CO) also exploring some of the links between the two. The approach adopted is algorithmic.
- Program: Introduction to Linear Programming (LP) [1] what is an LP problem [2] modeling your problem as a linear program [3] the simplex method (description and analysis) [4] duality theory [5] complementary slackness [6] economic interpretation [7] sensitivity analisys [8] geometric interpretation. Introduction to graphs and Combinatorial Optimization (CO) [1] graphs and digraphs as models [2] a few good characterizations (bipartite graphs, eulerian graphs, Planar Graphs, Chordal Graphs) [3] shortest paths [4] minimum spanning trees [5] max flows and min cuts [6] bipartite matching.
- Period: January, 22, 2014– February, 25, 2014, intensive course.
- WWW page of the course: `profs.sci.univr.it/~rrizzi/classes/MathProg`