Admission to the Master degree in Mathematics
Syllabi for the various Curricula

In the written examination applicants will be asked for definitions, examples and statements on the topics listed below. They will also be asked to solve simple exercises.

Curriculum Advanced Mathematics

Algebra and Geometry

Definition and elementary properties of groups, Abelian groups, rings and fields.
Group and ring homomorphisms. Subgroups and normal subgroups. Subrings and ideals. Prime ideals and maximal ideals.
Quotient groups and rings. Ring and group isomorphisms, isomorphism theorems.
Projective, affine and euclidean spaces. Linear subspaces. Conics and quadrics.
Metric and topological spaces; continuous maps. Subspaces, products and quotients.
Compactness, Connectedness, Hausdorff spaces.
Topological manifolds. Homotopy of continuous maps. Fundamental group.
Differentiable curves and surfaces in three-dimensional space.

Mathematical Analysis

Differential and Integral Calculus
Lebesgue measure and Hausdorff measures. Measurable functions. Lebesgue's theory of integration. Convergence theorems for integrals.
Fubini's theorem. Change of variables in multiple integrals. Gauss-Green formulas.
Sequences and Series of functions: pointwise and uniform convergence. Power series.
Fourier series: L^2 theory of convergence and pointwise convergence.
Ordinary differential equations
Graphical representation of solutions.
The Cauchy problem for ordinary differential equations: existence of local solutions, uniqueness and existence of maximal solutions. Existence and uniqueness of solutions of first order systems. Examples: Differential equations with separable variables: explicit computation of solutions of the Cauchy problem. Method of variation of arbitrary constants: explicit computation of solutions of the Cauchy problem for linear equations.
Two dimensional linear systems with constant coefficients.

Probability theory and statistics

- Basics. Conditional probability. Independence.
- Random variables. Definition, distribution. Standard examples.
- Limit theorems. Law of large numbers. Central limit theorem.

Mathematical Physics

-Basic aspects of Lagrangian formulation of classical Mechanics.
-Basic aspects of Hamiltonian formulation of classical Mechanics.
-Elementary theorems relating symmetries and constants of motion.
-Basic properties of D'Alembert Equation, Heat Equation, Poisson Equation and their solutions.

Curriculum Cryptography

Integers
Elementary properties of integers and rational numbers.
Euclidean algorithm for integers: the description of the algorithm, the related division theorem, the algorithm complexity. First applications of the Euclidean algorithm: integers represented in a non-decimal base, computation of greatest common divisors, Bézout identity, divisibility properties for integers, linear Diophantine equations.
Unique factorization for integers: fundamental theorem of arithmetic, irrationality proofs, factorization properties, least common multiple.
Prime numbers: Euclid's theorem, Prime Number Theorem, elementary properties of divisibility involving primes. Integer factorization: trial division, Eratosthenes' sieve.
Integer congruences: elementary properties of integer congruences, divisibility criteria, solution of linear congruence equations.
Congruence classes: congruences as equivalence relations, the arithmetic in Z/mZ, invertible elements in Z/mZ, Euler's phi function, solving linear equation in Z/mZ.
Partial fractions and continued fractions.

Groups and Rings
Definition and elementary properties of groups, Abelian groups and Rings.
Z and Z/mZ as groups and rings. Definition of fields and their elementary properties.
Multiplication in commutative rings: zero divisors, zero divisors in Z/mZ, cancellation rules, characterization of a field, when Z/mZ is a field, primitive element theorem.
Group homomorphisms and their elementary properties, kernel.
Ring homomorphisms and their elementary properties, kernel.
Group of units in a ring and its elementary properties.
Subgroups and normal subgroups. Subrings and ideals. Prime ideals and maximal ideals.
Cyclic groups, free groups. Cosets, Lagrange theorem and its inversion in the Abelian case.
Symmetric group, alternating group, general linear group, dihedral group, Cayley's theorem.
Quotient groups. The characteristic of a ring, (ring and group) isomorphisms, the three isomorphism theorems (group version and ring version).
Matrices with entries in rings. Order of elements (multiplicative, additive) in rings/groups, Cauchy's theorem. Abstract Fermat theorem.
Product of groups and product of rings. Cyclic decomposition of finite Abelian groups.

Z/mZ and special integers
Orders, Euler's theorem, fast computation of Euler's phi function, Fermat theorem, binomial theorem, the Frobenius map (also in rings with positive characteristic), computation of high powers.
Special numbers: pseudo-primes, Fermat's numbers, Mersenne's, Carmichael's.
The Chinese Remainder Theorem and congruence equation systems.
Roots of unity. Cyclic decomposition of Z/m. Quadratic residues, Legendre's (Jacobi's) symbol and the law of quadratic reciprocity.

Univariate Polynomials
Polynomials, monomial, terms: basic operations and definitions (coefficients, degree).

Polynomials over the integers, over a field, over a general commutative ring.
Unique factorization for univariate polynomials, irreducible polynomials, division theorem, Euclidean algorithm and Bézout's theorem. Divisibility properties.
Ruffini's theorem and D'Alembert's theorem. Relation between polynomials and their interpretation as functions.
The fundamental theorem of Algebra and its consequences. Rational functions, partial fractions.
Derivatives of polynomials and root multiplicity (any characteristics).
Properties of real polynomials and of rational polynomials. Gauss's lemma and Descartes's rational root theorem. Irreducibility criteria for rational polynomials (Eisenstein, etc..).
Polynomials over a field: Chinese Remainder Theorem, congruence equation systems, interpolation theorem. Factorization of integral polynomials.

Fields
Properties of the group of units F* of a field F: any finite subgroup is cyclic.
Special case: (Z/pZ)*. Primitive roots modulo m. Field extensions.
Finite fields: congruences classes modulo polynomials, extension of Z/pZ using polynomial quotients, arithmetic of elements in Z/pZ[x]/(f), the Cauchy-Kronecker-Steinitz theorem, splitting fields, simple extensions. The size of finite fields and the isomorphisms among them.

Computer Science
CPU's and memory. Compilers. Variables and state. Expressions and commands: syntax and semantics. Assignments, conditionals (if, then, case), cycles (while, for). Types of data: bits, bytes, integers, char, floating point numbers. Definition and call of a method/function: parameters, global variables, local variables. Recursive functions. Vectors: entry access, sum, product. File opening (reading/writing).

Curriculum Mathematics and Statistics for Life and Social Sciences

Probability theory and statistics
- Basic axioms and examples. Conditional probability. Independence.
- Random variables. Definition, distribution. Standard examples.
- Limit theorems. Law of large numbers. Central limit theorem.
- Markov chains with finite states.
- Estimators and estimate. Testing of statistical hypothesis: main concepts.

Mathematical Analysis
Differential and Integral Calculus
- Sequences and Series of functions: pointwise and uniform convergence. Power series.
- Fourier series: L^2 and pointwise convergence.

Differential equations
- Solutions of separable equations. Solutions of linear equations and linear systems with constant coefficients.
- Existence and uniqueness theorem for Cauchy problems.
- Vector field associated to a system of differential equations. Equilibria.

Numerical analysis
Systems of linear equations:
- Gauss elimination method.

Nonlinear equations:
- bisection method,
- Newton method,
- secant method,

Polynomial interpolation:
- Lagrange interpolator polynomial,
- compound interpolation (piecewise polynomial).

Numerical integration:
- simple and compound trapezoid formula.

Ordinary differential equations:
- Euler method.

Curriculum Teaching and Scientific Communication

Algebra. Definition and elementary properties of groups, Abelian groups, rings and fields. Group homomorphisms. Subgroups, normal subgroups and quotient groups.

Topology in R^n. Open sets, closed sets, compact sets. Closure and boundary of a set. Connected and simply connected sets.

Curves and surfaces. Basic notions and theorems about differentiable curves and surfaces in three-dimensional space.

Sequences and series of functions. Pointwise convergence, uniform convergence. Power series, radius of convergence. Pointwise convergence properties of the Fourier series.

Functions of several real variables. Continuity, differentiability. Weierstrass theorem. Mean value theorem of Lagrange. Higher order partial derivatives, Schwarz theorem. Taylor formula (first and second order) and application to the study of the shape of the graph. Implicit function theorem. Inverse function theorem.

Measure and Integral. Jordan content. Riemann integral. Fubini theorem. Curvilinear integrals of functions and vector fields. Definition of Lebesgue measure, definition of Lebesgue integral. Surface integrals of functions and vector fields. Gauss theorem, Green theorem and Stokes theorem.

Complex functions of one complex variable. Complex derivative. Cauchy-Riemann conditions. Integral of a complex function along an oriented curve.