



## **Admission to the Master degree in Mathematics**

### **Syllabus for Linear Algebra and Calculus Requested for every Curriculum**

In the written examination applicants will be asked for definitions, examples and statements on the topics listed below. They will also be asked to solve simple exercises.

#### **Linear algebra**

Vectors in three-dimensional space. Equations of lines and planes. Distances.

Matrices. Operations on matrices. Matrices and linear systems. Gaussian method for row reduction, Rouchè-Capelli's Theorem.

Vector spaces: subspaces, linear dependence and independence, generators, Grassmann formula. Bases and dimension of a vector space.

Determinant and rank: properties of the determinant, rank of a matrix.

Linear functions: kernel and image, rank-nullity theorem and its applications, representative matrices, similar matrices, change of basis.

Eigenvalues and eigenvectors: definitions and examples, characteristic polynomial, criteria for diagonalizability.

Scalar product: orthogonal projection, norm, distance, orthonormal bases, orthogonal complement. Symmetric endomorphisms. Bilinear and quadratic forms.

#### **Calculus**

Elements of set theory: logical operators, quantifiers. Basic operations of set theory. Relations, functions.

Induction principle. Numerical sets: natural, integers, rational, real and complex numbers.

*One variable functions.*

Limits of one variable functions: definition and properties. Sequences of real numbers: examples, limit of a sequence, Cauchy sequences. Continuous one variable functions: definition. The intermediate value theorem.

*Differential calculus for one variable functions.*

Derivative and differential. The theorems of Fermat, Rolle, Lagrange e de l'Hospital. Taylor formula. Graph of a function.

*Riemann integral for one variable functions.*

Definition and basic properties. Fundamental theorem of integral calculus. Integral functions. Integration techniques. Numerical series: definition of convergence and criteria for convergence. Improper integrals.

*Functions in several variables.*

Topology in  $\mathbb{R}^n$ : internal, external and boundary points. Open, closed and bounded subsets. Bounded and closed sets are sequentially compact. Limits for functions of several variables.

Continuity for functions of several real variables. Directional and partial derivatives. Differentiable functions.

Higher order derivatives. Implicit function theorem.