SCHOOL OF DOCTORAL STUDIES IN ENVIRONMENTAL ENGINEERING - XIX cycle

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High Order ADER FV/DG Methods for Hyperbolic Equations

Abstract of the doctoral thesis

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We are concerned with development of high order numerical methods for hyperbolic equations. We construct ADER Finite Volume (FV) and ADER Discontinuous Galerkin (DG) numerical methods on non-structured triangular meshes.

The high order character of the methods is obtained using the ADER approach where polynomial representation of the data in space is a first requirement. In the case of ADER-FV schemes the space representation of the data is obtained by a suitable reconstruction of the data. For ADER-DG schemes the high order space representation is naturally introduced with the use of basis functions. For both approaches one solves the Derivative Riemann Problem (DRP), for which the starting point is a Taylor's series expansion of its solution at the element interface, as a function of time.

In this thesis we propose new techniques for solving the DRP approximately and compare their performance with the existing DRP solver of Toro and Titarev (2002, 2006). For the space reconstruction procedure we implement the technique presented by Dumbser and Käser (2006).

We apply the numerical methods to the single-phase gas dynamic system, the Euler equations, supplemented by a general equation of state (Stiffened gas and Van der Waals), and to the Shallow Water system. A numbers of test problems are implemented in order to assess the numerical methods, verifying the expected order of convergence.

Beside this, we study multiphase flow models. The isentropic model presented by Saurell-Abgrall (1999) is studied and an approximate Riemann solver is proposed (Castro and Toro 2006). Furthermore, two Riemann solvers are proposed to solve the multiphase model presented by Baer-Nunziato (M. Baer and J. Nunziato 1986). We extend the EVILIN solver (Toro, 2006), currently available for single-phase flow problems, as applied to the full two-phase flow system. In addition we use the stratification hypothesis in order to decouple the two-phase system into two single-phase Euler systems. Finally, we extended the exact Riemann solver presented by Schwendeman et al. (2006) to a general equation of state solving an algebraic non-linear system of five equations.

All of these Riemann solvers can be used as a leading term in the DRP solution to produce ADER numerical schemes.