



PhD in Mathematics – Cycle 37
Research topics 2021

Bando 2021/Call 2021

Dipartimenti di Eccellenza/Departments of Excellence
Assegni di ricerca/Fellowship

A	Modelling dynamic networks with differential equations
B	Mathematics of Reinforcement Learning
C	Geometric Analysis

Fondazione Bruno Kessler - FBK
Borsa di studio/ Scholarship

D	Data-driven approaches in epidemiological modeling
E	Evaluation of interventions against infectious diseases
F	Analytical, stochastic, and applicative aspects of Deep Neural Networks

Eustema S.p.A
Borsa di studio/ Scholarship

G	Blockchain technology for digital notary
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PAT (AI) Laboratorio Q@TN
Borsa di studio/ Scholarship

H	Development and security proof of Quantum Key Distribution protocols based on single particle entanglement (SP-QKD)
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Assegno di ricerca/Fellowship A

Topic: Modelling dynamic networks with differential equations
P.I.: Veronica Vinciotti
Contacts: Veronica Vinciotti (veronica.vinciotti@unitn.it)
<p>Synthetic description of the activity and expected research outcome</p> <p>Many processes, such as the spread of an infectious disease or the differentiation of stem cells to more specific cell types, live on an underlying network which evolves stochastically over time. Stochastic quasi-reaction models provide a generic framework for describing the stochastic nature of these complex processes, i.e. how the state of the system changes from its current state. These changes can be affected also by external factors which may drive the network process, such as face-mask compliance.</p> <p>When the objective is to infer the underlying dynamics and detect important drivers from observational network data, many statistical challenges arise: the network or the external factors may be noisy or only partially observed, the system may be made by a very large number of components or affected by many factors and there may be heterogeneity among the observations, such as at the level of the individual node.</p> <p>In this PhD project, we will investigate these challenges both from a methodological and a computational point of view, tackling fundamental issues underlying statistical inference for dynamic network modelling as well as looking at the specifics of certain applications, where more bespoke models may be needed. The project will be co-supervised by Prof. Ernst Wit from the Università della Svizzera Italiana, with whom the student will be able to spend a period of their PhD time.</p>
<p>References</p> <p>[1] Biasco, L., et al. "In vivo tracking of human hematopoiesis reveals patterns of clonal dynamics during early and steady-state reconstitution phases." <i>Cell stem cell</i> 19.1 (2016): 107-119.</p> <p>[2] Pellin, D. et al. "Penalized inference of the hematopoietic cell differentiation network via high-dimensional clonal tracking." <i>Applied Network Science</i> 4.1 (2019): 1-26.</p> <p>[3] Užupytė, R and E. Wit. "Test for triadic closure and triadic protection in temporal relational event data." <i>Social Network Analysis and Mining</i> 10.1 (2020): 1-12.</p> <p>[4] Wilkinson, D. (2018) <i>Stochastic Modelling for Systems Biology</i>. CRC Press.</p> <p>[5] Wood, S and E. Wit. "Was $R < 1$ before the English lockdowns? On modelling mechanistic detail, causality and inference about Covid-19." <i>medRxiv</i> (2021).</p>
<p>Ideal candidate (skills and competencies)</p> <p>The ideal candidate is a student with a Master's degree (or similar) in mathematics, statistics, physics or computer science with excellent mathematical background and strong knowledge in probability and statistics. Good computer skills, such as knowledge of computer languages. Fluent written and spoken English.</p>

Assegno di ricerca/Fellowship B

Topic: Mathematics of Reinforcement Learning
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Synthetic description of the activity and expected research outcome

Machine learning is a very active interdisciplinary research topic. In the past few years, its importance has become apparent for the general public, too, as more and more applications find their place in solving real-world problems. At the same time, the initial approach, verging on the experimental side, started showing its limitations, and there is an increasing interest in a solid theoretical understanding. This comes not only from the purely mathematical perspective of proving results but from the more general need of preventing bias or unexpected consequences of poorly understood techniques and algorithms.

Reinforcement Learning is a subset of machine learning in which an agent interacts with an environment, acting and observing the consequences of its actions thus learning how to solve decision problems, with goals depending on the state of the environment [4].

Reinforcement learning is currently an extremely active field and results considered impossible just a few years ago are quite frequent. In 2020, Agent57 was the first deep reinforcement learning agent to outperform humans in all of the 57 games in the Atari benchmark [1].

This benchmark includes very different games that the agent had no previous knowledge of, so that it requires a good degree of flexibility to succeed, given the varying reward structures. Also in 2020, a paper on the MuZero algorithm was published in Nature [2]. MuZero learned how to beat humans in 51 of the 57 Atari games, but also to play go, chess and shogi as well as AlphaGo, another algorithm that, unlike MuZero, had been provided with information about the games, namely their rules. These examples come from (video)games, but this does not diminish the importance of the result, as Go, for example, was seen up to a few years ago, as the prototype of the task in which humans would be better than machines for a long time. Moreover, there are also applications to other fields, such as computer science and finance.

Some of the peculiarities of Reinforcement Learning, compared to other areas of machine learning, include the absence of information of the consequences of actions, forcing trial and error strategies, and the delayed nature of the rewards. This leads to the competing necessities of figuring out which are the possible actions (exploration) and gathering the rewards that already discovered actions provide (exploitation). If the agent settles for the rewards of the first actions discovered, it risks missing out more efficient ones. On the other hand, if the focus is on exploration, it might never settle on a reward and, if the environment is rich enough, could find no strategy.

As already mentioned, the picture is even more challenging due to the several sources of uncertainty such as partial knowledge of the environment and of the impact of the actions taken, because of the delay in the reward. At the same time, these characteristics are strengths of the agent, as it does not have any domain-specific knowledge and its strategy (or policy) can be used in different settings.

Following a general trend in machine learning, there is an increased effort to understand the mathematics behind some of the techniques used, to apply and generalise them in a thoughtful fashion and, eventually, to have a sound understanding of their capabilities and their limitations.

The project will focus on solving theoretical problems in this framework, such as (but not limited to) the ones described in [3].

Let us see some examples. The Sarsa (l) algorithm, together with linear approximators has been successfully used to find an optimal policy. In general, the convergence does not hold, however, from an empirical point of view, the algorithm oscillates around a good policy. Is it possible to characterise properly that solution? Which are its properties? Is it possible to provide theoretical upper bounds on the distance between such a solution and the optimal policy?

The combination of a single bootstrapping algorithm, off-policy methods and approximation is known in the field as the deadly triad, as it generally leads to divergence [5]. However, it is still unclear, from a theoretical point of view, what are the reasons behind this phenomenon.

References

- [1] Adrià Puigdomènech Badia, Bilal Piot, Steven Kapturowski, Pablo Sprechmann, Alex Vitvitskyi, Daniel Guo, and Charles Blundell. Agent57: Outperforming the Atari Human Benchmark. ArXiv:2003.13350 [cs, stat], 2020.
- [2] Julian Schrittwieser, Ioannis Antonoglou, Thomas Hubert, Karen Simonyan, Laurent Sifre, Simon Schmitt, Arthur Guez, Edward Lockhart, Demis Hassabis, Thore Graepel, Timothy Lillicrap, and David Silver. Mastering Atari, Go, chess and shogi by planning with a learned model. Nature, 588(7839):604 609, 2020.
- [3] Richard S. Sutton. Open Theoretical Questions in Reinforcement Learning. In G. Goos, J. Hartmanis, J. van Leeuwen, Paul Fischer, and Hans Ulrich Simon, editors, Computational



Learning Theory, volume 1572, pages 11–17. Springer Berlin Heidelberg, Berlin, Heidelberg, 1999.

- [4] Richard S. Sutton and Andrew G. Barto. Reinforcement Learning: An Introduction. Adaptive Computation and Machine Learning Series. The MIT Press, Cambridge, Massachusetts, Second edition edition, 2018.
- [5] Hado van Hasselt, Yotam Doron, Florian Strub, Matteo Hessel, Nicolas Sonnerat, and Joseph Modayil. Deep Reinforcement Learning and the Deadly Triad. ArXiv:1812.02648 [cs], 2018.

Ideal candidate (skills and competencies)

The ideal candidate for this PhD position has a strong mathematical background, in particular in Probability and Statistics. Good knowledge of algorithms and programming skills are desirable, but not necessary. The candidate will conduct research in a rapidly evolving field, rich with challenging and interesting problems, under the supervision of Prof. Claudio Agostinelli and Dr. Luigi Amedeo Bianchi, and the external collaboration of Prof. Maurizio Parton of the University of Chieti- Pescara.

Assegno di ricerca/Fellowship C

Topic: **Geometric Analysis**

P.I.: **Lorenzo Mazzieri**

Contacts: **Lorenzo Mazzieri** (lorenzo.mazzieri@unitn.it)

Synthetic description of the activity and expected research outcome

The fellowship is meant for candidates with a solid background in Analysis and Geometry, and will be devoted to the study of the geometric aspects of elliptic partial differential equations, with special attention to methods and techniques coming from Riemannian geometry. The main focus will be on the following two aspects:

- 1) Extensions of the range of applicability of the present techniques, possibly with the introduction of new methods.
- 2) Applications to the study of the qualitative properties and to the classification of solutions to overdetermined elliptic boundary value problems arising in the field of shape optimization, in mathematical relativity (static metrics), and in the study of geometric flows (self-similar solutions).

References

- [1] V. Agostiniani M. Fogagnolo and L. Mazzieri. Sharp geometric inequalities for closed hypersurfaces in manifolds with nonnegative Ricci curvature. *Inventiones*, volume 222, pages 1033–1101 (2020).
- [2] V. Agostiniani and L. Mazzieri. On the Geometry of the Level Sets of Bounded Static Potentials. *Comm. Math. Phys.*, 355(1):261–301, 2017
- [3] V. Agostiniani and L. Mazzieri. Monotonicity formulas in potential theory. *Calculus of Variations and Partial Differential Equations* 59, Article number: 6 (2020).
- [4] L. Ambrozio. On static three-manifolds with positive scalar curvature. *J. Differential Geom.*, 107(1):1–45, 2017.
- [5] S. Borghini and L. Mazzieri. On the mass of static metrics with positive cosmological constant: II. *Comm. Math. Phys.*, 377(3):2079–2158, 2020.
- [6] T. H. Colding. New monotonicity formulas for Ricci curvature and applications. I. *Acta Mathematica*, 209(2):229–263, 2012.
- [7] T. H. Colding and W. P. Minicozzi. Monotonicity and its analytic and geometric implications. *Proceedings of the National Academy of Sciences*, 110(48):19233–19236, 2013.
- [8] T. H. Colding and W. P. Minicozzi. Ricci curvature and monotonicity for harmonic functions. *Calculus of Variations and Partial Differential Equations*, 49(3):1045–1059, 2014.
- [9] G. Crasta, I. Fragalà, and F. Gazzola. On a long-standing conjecture by Pólya-Szegő and related topics. *Z. Angew. Math. Phys.*, 56(5):763–782, 2005.



- [10] Farina, L. Mari, and E. Valdinoci. Splitting theorems, symmetry results and overdetermined problems for Riemannian manifolds. *Comm. Partial Differential Equations*, 38(10):1818–1862, 2013.
- [11] G. Huisken and T. Ilmanen. The inverse mean curvature flow and the Riemannian Penrose inequality. *J. Differential Geom.*, 59(3):353–437, 2001.

Ideal candidate (skills and competencies)

The ideal candidate should have obtained a Master degree in Mathematics, or in another scientific field (Physics, Engineering...) but with a solid mathematical background, especially in the area of differential equations, and a strong interest in modelling applied problems. A good knowledge of probability and statistics, and experience in scientific programming are a plus.

Borsa di studio/ scholarship D

Topic: **Data-driven approaches in epidemiological modeling**

P.I.: **Andrea Pugliese and Piero Poletti**

Contacts: **Andrea Pugliese** (andrea.pugliese@unitn.it)

Synthetic description of the activity and expected research outcome:

Research activity conducted during the Ph.D. will focus on the development of epidemiological models informed by real-world data aimed at investigating the main determinants of the disease spread in humans. Envisioned approaches range from the study of mechanistic models mimicking the spatio-temporal transmission dynamics of infectious diseases to the use of Bayesian approaches applied to detailed epidemiological records.

References

- [1] Estimating sources and sinks of malaria parasites in Madagascar. Ihantamalala FA, Herbreteau V, Rakotoarimanana FMJ, Rakotondramanga JM, Cauchemez S, Rahoilijaona B, Pennober G, Buckee CO, Rogier C, Metcalf CJE, Wesolowski A. *Nat Commun.* 2018 Sep 25;9(1):3897. doi: 10.1038/s41467-018-06290-2.
- [2] Heterogeneity in social and epidemiological factors determines the risk of measles outbreaks. Bosetti P, Poletti P, Stella M, Lepri B, Merler S, De Domenico M. *Proc Natl Acad Sci U S A.* 2020 Dec 1;117(48):30118-30125. doi: 10.1073/pnas.1920986117.
- [3] Spatiotemporal spread of the 2014 outbreak of Ebola virus disease in Liberia and the effectiveness of non-pharmaceutical interventions: a computational modelling analysis. Merler S, Ajelli M, Fumanelli L, Gomes MF, Piontti AP, Rossi L, Chao DL, Longini IM Jr, Halloran ME, Vespignani A. *Lancet Infect Dis.* 2015 Feb;15(2):204-11. doi: 10.1016/S1473-3099(14)71074-6.
- [4] The impact of COVID-19 and strategies for mitigation and suppression in low- and middle-income countries. Walker PGT, Whittaker C, Watson OJ, Baguelin M, Winskill P, Hamlet A, Djafaara BA, Cucunubá Z, Olivera Mesa D, Green W, Thompson H, Nayagam S, Ainslie KEC, Bhatia S, Bhatt S, Boonyasiri A, Boyd O, Brazeau NF, Cattarino L, Cuomo-Dannenburg G, Dighe A, Donnelly CA, Dorigatti I, van Elsland SL, FitzJohn R, Fu H, Gaythorpe KAM, Geidelberg L, Grassly N, Haw D, Hayes S, Hinsley W, Imai N, Jorgensen D, Knock E, Laydon D, Mishra S, Nedjati-Gilani G, Okell LC, Unwin HJ, Verity R, Vollmer M, Walters CE, Wang H, Wang Y, Xi X, Lalloo DG, Ferguson NM, Ghani AC. *Science.* 2020 Jul 24;369(6502):413-422. doi: 10.1126/science.abc0035.

Ideal candidate (skills and competencies)

Ideal candidates will have a degree in applied mathematics, statistics or related areas, a good knowledge of basic statistical methods and of scientific programming; experience in mathematical modelling would be a plus.



Borsa di studio/ scholarship E

Topic: Evaluation of interventions against infectious diseases
P.I.: Andrea Pugliese and Giorgio Guzzetta
Contacts: Andrea Pugliese (andrea.pugliese@unitn.it)
<p>Synthetic description of the activity and expected research outcome</p> <p>The Ph.D. student will evaluate control measures against different infectious diseases, both retrospectively and prospectively, by developing mathematical models calibrated against observed epidemiological data and informed by other data relevant to the infection under study. The modelling approach will be tailored to the addressed problems and may include compartmental models, generative models, individual-based simulations as well as Bayesian approaches and will include scenario analysis to compare alternative intervention strategies.</p>
<p>References</p> <p>[1] Retrospective analysis of the Italian exit strategy from COVID-19 lockdown. Marziano V, Guzzetta G, Rondinone BM, Boccuni F, Riccardo F, Bella A, Poletti P, Trentini F, Pezzotti P, Brusaferrero S, Rezza G, Iavicoli S, Ajelli M, Merler S. Proc Natl Acad Sci U S A. 2021 Jan 26;118(4):e2019617118. doi: 10.1073/pnas.2019617118.</p> <p>[2] Early assessment of diffusion and possible expansion of SARS-CoV-2 Lineage 20I/501Y.V1 (B.1.1.7, variant of concern 202012/01) in France, January to March 2021. Gaymard A, Bosetti P, Feri A, Destras G, Enouf V, Andronico A, Burrel S, Behillil S, Sauvage C, Bal A, Morfin F, Van Der Werf S, Josset L; ANRS MIE AC43 COVID-19; French viro COVID group, Blanquart F, Coignard B, Cauchemez S, Lina B. Euro Surveill. 2021 Mar;26(9):2100133. doi: 10.2807/1560-7917.ES.2021.26.9.2100133.</p> <p>[3] Parental vaccination to reduce measles immunity gaps in Italy. Marziano V, Poletti P, Trentini F, Melegaro A, Ajelli M, Merler S. Elife. 2019 Sep 3;8:e44942. doi: 10.7554/eLife.44942.</p> <p>[4] Containing Ebola at the Source with Ring Vaccination. Merler S, Ajelli M, Fumanelli L, Parlamento S, Pastore Y Piontti A, Dean NE, Putoto G, Carraro D, Longini IM Jr, Halloran ME, Vespignani A. PLoS Negl Trop Dis. 2016 Nov 2;10(11):e0005093. doi: 10.1371/journal.pntd.0005093.</p>
<p>Ideal candidate (skills and competencies)</p> <p>Ideal candidates will have a degree in applied mathematics or related areas, some experience in mathematical modelling, and a good competence of scientific programming; knowledge of basic statistical methods would be a plus.</p>

Borsa di studio/ scholarship F

Topic: Analytical, stochastic, and applicative aspects of Deep Neural Networks
P.I.: Gian Paolo Leonardi
Contacts: Gian Paolo Leonardi (gianpaolo.leonardi@unitn.it)
<p>Synthetic description of the activity and expected research outcome</p> <p>The aim of the proposed research is twofold. The theoretical part of the research will concern the mathematical aspects of artificial neural networks, including stochastic and measure-theoretic representations, layer-wise regularization methods, expressivity and asymptotic properties of special classes of deep neural networks (like, e.g., quantized neural networks). The experimental part of the research, conducted at the Data Science for Health (DSH) Lab of the Fondazione Bruno Kessler, will focus on Digital Pathology challenges addressed by deep learning solutions. The expected research</p>



outcomes include: the development of new mathematical tools for more efficient and robust approximation & training of general DNN architectures, with special emphasis to quantization; the implementation and testing of new algorithms that may significantly reduce computational costs without affecting performance, robustness and interpretability.

References

- [1] G. P. Leonardi and M. Spallanzani. Analytical aspects of non-differentiable neural networks. arXiv preprint arXiv:2011.01858, 2020.
- [2] N. Bussola, A. Marcolini, V. Maggio, G. Jurman, and C. Furlanello. Ai slipping on tiles: data leakage in digital pathology. In Proceedings of International Workshop on Artificial Intelligence for Digital Pathology 2021 (AIDP), volume 12661 of Lecture Notes in Computer Science, pages 167–182. Springer International Publishing, 2021.
- [3] Y. Bengio. Learning deep architectures for AI. Foundations and Trends in Machine Learning, 2:1–127, 2009.
- [4] I. Hubara, M. Courbariaux, D. Soudry, R. El-Yaniv, and Y. Bengio. Quantized neural networks: training neural networks with low precision weights and activations. Journal of Machine Learning Research, 18:1–30, 2018.
- [5] J. Lee, Y. Bahri, R. Novak, S. Schoenholz, J. Pennington, and J. Sohl-Dickstein. Deep neural networks as gaussian processes. In International Conference on Learning Representations, 2018.
- [6] J. Su, M. Cvitkovic, and F. Huang. Sampling-free learning of bayesian quantized neural networks. arXiv preprint arXiv:1912.02992, 2019.

Ideal candidate (skills and competencies)

The ideal candidate should have a good background in Analysis, Probability and Linear Algebra, as well as good programming skills in Python, with at least basic knowledge of a modern DL framework such as TensorFlow and/or Pytorch. She/He should be strongly committed towards both the theoretical and the applied aspects of the research.

Borsa di studio/ scholarship G

Topic: Blockchain technology for digital notary
P.I.: Massimiliano Sala
Contacts: Massimiliano Sala (massimiliano.sala@unitn.it)
Synthetic description of the activity and expected research outcome Blockchain technology is an emergent field in applied cryptography and one of its main applications is digital notary. The PHD student will research cryptographic primitives and protocols that guarantee security in the blockchain application.
References [1] Alessio Meneghetti, Massimiliano Sala and Daniele Taufer, “A Survey on PoW based Consensus”, Annals of Emerging Technologies in Computing (AETiC), Print ISSN: 2516-0281, Online ISSN: 2516-029X, pp. 8-18, Vol. 4, No. 1, 2020, Available: http://aetic.theiaer.org/archive/v4/v4n1/p2.html
Ideal candidate (skills and competencies) The ideal candidate has a deep understating of pure and applied cryptography, as well as familiarity with blockchain technology and its applications



Borsa di studio/ scholarship H

Topic: **Development and security proof of Quantum Key Distribution protocols based on single particle entanglement (SP-QKD)**

P.I.: **Sonia Mazzucchi, Silvio Ranise, Lorenzo Pavesi**

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Synthetic description of the activity and expected research outcome

The PhD student will be enrolled in the transdisciplinary program in Quantum Sciences and Technologies <https://www.unitn.it/drphys/en/421/transdisciplinary-program-quantum-science-and-technologies>

The main goal of the PhD student's thesis work is the development, the security analysis as well as a critical evaluation of new quantum key distribution protocols based on single-particle entanglement (SPE) [Az20]. Experiments will also be performed to validate the proposed QKD protocol. The project acronym is SP-QKD (which stands for Security Proof of QKD).

Since one of the main issues affecting the security of QKD is the authentication of the two clients and the possibility of a "man in the middle attack", we plan to tackle this problem. On the one hand, we plan to propose some use-cases where the high security promised by QKD protocols is not significantly damaged by authentication issues. On the other hand, we plan to develop and analyze new protocols more robust under man-in-the-middle attacks by introducing a trusted arbiter, similarly to what is done in [Ma19]. Eventually, the developed protocols will be implemented with a fiber optics or a free space based set-up with the SPE sources available in the laboratories of the NanoLab of the physics department. This will allow us to verify the protocol at different levels of abstraction, namely on design with possibly automated security analyses and on implementation with an experimental assessment of performance and security risks. Here, the PhD student will collaborate in assessing and evaluating the results of the analyses at the various levels of abstractions.

References

- [1] [Az20] S.Azzini, S. Mazzucchi, V. Moretti, D. Pastorello, L. Pavesi. Single-particle Entanglement.. -Advanced Quantum Technologies ISSN 2511-9044. - 2020 vol 3:10(2020).
- [2] [Ma19] F. Massa, Francesco et al. Experimental two-way communication with one photon. Advanced Quantum Technologies, 2019, 2.11: 1900050.

Ideal candidate (skills and competencies)

The PhD student enrolled in this project is assumed to be mainly a mathematician with an expertise in modern cryptography and security analysis. He/she will mainly work at the department of mathematics and FBK, but he/she should be also able to collaborate with experimental physicists of the Nanoscience Laboratory based at the department of physics.