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A New 3D Parallel SPH scheme for Free Surface Flows

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Abstract

The focus of the research activity is on the Smoothed Particle Hydrodynamics (SPH) method and its applications to environmental flow problems. The purpose is to model fully three-dimensional non-hydrostatic free surface flows and impact problems with the SPH method applying it to inviscid and viscous fluids. After a thorough literature review of the SPH method, the research work is divided up into two different parts.

First, we start analysing the numerical stability for two different SPH schemes, one is based on the approach of Ben Moussa and Vila, another one is proposed in this work and mimics the classical 1D Lax-Wendroff scheme. In both approaches the classical artificial viscosity term of Monaghan is removed, since artificial viscosity requires a careful calibration of several parameters depending on the test cases. Nevertheless, the linear stability of the new methods has been preserved, as demonstrated via a von Neumann stability analysis. Moreover, the issue of the consistency for the equations of gas dynamics is analyzed. An alternative approach is proposed that consists of using Godunov-type SPH schemes in Lagrangian mass coordinates. This does not only provide an improvement in accuracy of the numerical solutions, but also assures that the consistency condition on the gradient of the kernel function is satisfied during the whole simulation for an initially equidistant particle distribution in Lagrangian mass coordinates. Three different Riemann solvers are implemented for the first-order Godunov-type SPH schemes in Lagrangian coordinates, namely the Godunov flux based on the exact Riemann solver, the Rusanov flux and a new modified Roe flux, following the work of Munz. Some well-known 1D shock tube test cases are solved and very satisfactory results have been obtained. The comparison of the numerical solutions computed by the Godunov-type SPH schemes in Lagrangian coordinates with the first-order Godunov finite volume method in Eulerian coordinates and the standard SPH scheme with Monaghan's viscosity term proves a significant improvement in the SPH accuracy. However, the extension of the SPH scheme in Lagrangian mass coordinates to multiple space dimensions is very challenging and therefore we develop an alternative approach using Lagrangian schemes in Eulerian coordinates.

Second, we propose a new robust and accurate SPH scheme for weakly compressible non-hydrostatic three-dimensional free-surface flows. The key idea of the new approach is the use of a monotone upwind flux in the semi-discrete form of the

density equation. In our particular implementation, we choose the simple and robust Rusanov flux. The monotone upwind flux, which does not contain any parameter that has to be tuned, automatically stabilizes the density field, and as a consequence it also stabilizes the pressure field, which is computed by the standard Tait equation of state. In contrast to the traditional SPH approach of Monaghan, which contains an artificial viscosity term, we use a simple centered discretization of the pressure terms in the velocity equation. This is also physically justified, since pressure waves do not have any preferred direction. After these modifications with respect to the classical SPH method, our new SPH scheme therefore recalls the advection-upstream-splitting-method (AUSM) of Liou, developed in the finite volume framework, which also uses a centered discretization of the pressure terms together with an upwind method for the convection. Now, since the SPH method is a Lagrangian scheme, the convection terms do not appear explicitly because they are already included in the material derivatives of the flow quantities. We note that our new SPH formulation can be either used in the original formulation of Monaghan or in the more recent SPH formulation of Ben Moussa and Vila, where one simply has to exchange the Riemann solver. So far, we considered only the semi-discrete case. For the fully discrete scheme, i.e. including the time discretization, further stabilization is obtained using a third order accurate TVD Runge-Kutta time stepping scheme, as successfully proposed by Shu and Osher in the context of high order essentially non-oscillatory finite volume and finite difference schemes for hyperbolic conservation laws.
To asses the accuracy of the new SPH scheme, a 3D mesh-convergence study has been performed for the strongly deforming free surface in a complex three-dimensional dam-break and wave-impact test problem. Very good results have been obtained and the mesh-convergence of the method has been verified for the continuous part of the flow even for this highly non-trivial test case. An improvement of the non-penetration boundary condition at the solid interfaces for the SPH particles has also been carried out. A new flexible approach to impose the boundary conditions at solid walls is proposed. It is flexible, accurate and able to handle any moving rigid body with arbitrarily irregular geometry. It does neither produce oscillations in the fluid pressure in proximity of the interfaces, nor does it have a restrictive impact on the stability condition of the explicit time stepping method, unlike the repellent boundary forces of Monaghan. As a further benefit, it does not contain any parameter that has to be calibrated.
Moreover, our new 3D SPH method has been implemented in parallel using the standard Message Passing Interface (MPI) paradigm together with a dynamic load-balancing strategy to improve the computational efficiency of the scheme. The dynamic redistribution of particles amongst the CPUs is a very crucial point in order to obtain an efficient scheme and is done calling the standard library METIS for graph and domain decomposition. Thus, simulations involving millions of fluid and solid particles can be run on modern massively parallel supercomputers in order to simulate large-scale environmental problems, which are the final aim of our research activity. A very good parallel performance has been achieved, as confirmed by a speed-up analysis. All 3D simulations have been carried out on two of the most powerful High Performance Computing (HPC) facilities in Europe, namely the Bundeshöchstleistungsrechenzentrum (HLRS) at the University of Stuttgart and the HLRB-II supercomputer at the Leibniz-Rechenzentrum of the Bavarian Academy of Sciences in München, Germany.
The numerical applications consist of environmental flow problems, such as dam-break and impact flows. The SPH solutions computed by the new parallel 3D SPH code have been compared with either experimental results or with other numerical reference solutions, obtaining in all cases a very satisfactory agreement.

Finally, the focus was on a catastrophic mudflow, that really happened in 1985 in Stava, Italy. This simulation is a big challenge even for sophisticated numerical methods because a very complex geometry has to be discretized, requiring a tremendous effort in terms of computer memory and also very large CPU times. From the physical point of view, the mudflow simulation contains also a lot of difficulties, due to the complexity of the break evolution of the tailing dams and the involved non-Newtonian fluid behaviour of the mud, as well as the development of turbulence. Millions of fluid and solid particles have been used to discretize the problem. The SPH results have been compared with the field measurements reported in published scientific contributions obtaining reasonable qualitative agreement.

RESEARCH OUTPUTS

1. Ferrari A., Dumbser M., Toro E. F., Armanini A., "A new stable SPH scheme in Lagrangian coordinates". *Communications in Computational Physics*, 2008, v. 4, n. 2, p. 378-404, ISSN: 1815-2406
2. Ferrari A., Dumbser M., Toro E. F., Armanini A., "A New Stable and Consistent Version of The SPH Method In Lagrangian". *ERCOFTAC Bulletin*, 2008, in press.
3. Ferrari A., Dumbser M., Toro E. F., Armanini A., "A new 3D parallel SPH scheme for free surface flows". *Computers and Fluids*, submitted to.
4. Ferrari A., Dumbser M., Toro E. F., Armanini A., "A New Stable and Consistent Version of the SPH Method in Lagrangian Coordinates". Proceedings of "SPHERIC: 2nd International Workshop", Universidad Politecnica de Madrid, 23rd-25th May 2007, ISBN: 978-84-690-6159-6.
5. Ferrari A., Dumbser M., Toro E. F., Armanini A., "A new 3D parallel SPH scheme for free surface flows". Proceedings of "ERCOFTAC SIG SPHERIC: 3rd International Workshop", EPFL EPFL-University Polytechnic of Lausanne, Switzerland, 4th-6th June 2008.
6. Ferrari A., Dumbser M., Toro E. F., Armanini A., Marx W., Wieprecht S."A New SPH Method for Landslide and Debris Flow Modelling". Report on HPC-EUROPA Project at High Performance Computing Center Stuttgart (HLRS) of the University of Stuttgart, Germany, 2008.