Courses of the Ph.D. Program in Mathematics, a.y. 2014/15

1. Algebra and mathematical logic

Title of the Course: **Dynamical Methods in Constructive Algebra**
Lecturer: Ihsen YENGUI, University of Sfax, (Tunisia)
Abstract: Constructive algebra [LQ,Y] can be seen as an abstract version of computer algebra. In computer algebra one attempts to construct efficient algorithms for solving concrete problems given in an algebraic formulation. Constructive algebra, on the other hand, can be understood as a first "preprocessing" step for computer algebra that leads to the discovery of general algorithms, though they are sometimes not efficient a priori. In constructive algebra, moreover, one tries to give general algorithms for solving virtually any theorem of abstract algebra.
A typical example is the theorem "any polynomial P in K[X] is a product of irreducible polynomials (K a field)". It seems as if there is no general algorithm that produces the irreducible factors. A possible way out is as follows: when performing computations with P, proceed as if its decomposition into irreducible polynomials were known. When something strange happens (e.g., when the gcd of P and another polynomial Q is a strict divisor of P), use this fact to improve the decomposition of P.
This method was invented in computer algebra as the D5-philosophy [D5], and later taken up in the form of the dynamical proof method in algebra [CLR].
E.g., it enables one to carry out computations inside the algebraic closure of K even though it is not possible to effectively construct this object, since in general this would require transfinite methods such as the Zorn Lemma.
[CLR] Coste M., Lombardi H., Roy M.-F. Dynamical method in algebra: Effective Nullstellensätze. Annals of Pure and Applied Logic 111 (2001) 203-256.
[D5] Della Dora J., Dicrescenzo C., Duval D. About a new method for computing in algebraic number fields. In Caviness B.F. (Ed.) EUROCAL '85. Lecture Notes in Computer Science 204, 289--290. Springer (1985).
[Y] I. Yengui. Lectures on Constructive Algebra---Projective modules over polynomial rings and dynamical Gr\"obner bases. Lecture Notes in Mathematics, Springer, to appear. (Preprint will be made available to students.)
[LQ] H. Lombardi and C. Quitt´e, Alg`ebre commutative. M´ethodes
constructives. Modules projectifs de type fini. Paris: Calvage & Mounet,
2012. English version forthcoming at Springer. (Preprint will be made available to students.)
30 hours

Title of the Course: **Set theory** (LM Matematica Trento)
Lecturer: Stefano Baratella
Obiettivi Formativi - introduzione alla teoria degli insiemi di Zermelo-Fraenkel con enfasi sui suoi legami con la pratica matematica.
Contenuti del Corso - Gli assiomi della teoria degli insiemi di Zermelo-Fraenkel. Buoni ordini ed ordinali. Gli ordinali classificano i buoni ordini. Equivalenti dell'assioma di scelta. Numeri cardinali. Aritmetica cardinale. Esempi di asserzioni relative alla pratica matematica che sono indipendenti da ZFC. L'estendibilita` della misura di Lebesgue a tutti i sottoinsiemi di [0,1] implica l'esistenza di un cardinale debolmente inaccessibile al di sotto della cardinalita`del continuo. Introduzione a risultati di consistenza relativa. Gli insiemi costruibili. Consistenza di AC relativamente a ZF.
Testi di riferimento - Kunen K., Set theory, College Publications. Jech T., Set theory -- The 3rd millennium edition, Springer. Devlin K., The joy of sets, Springer.
Period: February 16, 2015–April 30, 2015
30 hours

Students may attend only one course in the area of Algebra and Mathematical Logic.

2. *Geometry*

Title of the Course: **Toric geometry**
Lecturer: Luis E. Solá Conde
Outline: Toric geometry studies a special type of algebraic varieties that can be described in terms of discrete –combinatorial– data. Though small and manageable, the category of toric varieties is rich enough to be used to illustrate many of the basic concepts of algebraic geometry: affine and projective varieties, group actions, smoothness and singularity, divisors, line and vector bundles, etc. The course is meant as an introduction to algebraic geometry for PhD and Master students, for which we assume familiarity with undergraduate geometry, basic algebraic structures and complex numbers.
Program:
(1) Affine Varieties. Affine toric varieties. Cones and lattices.
(2) Abstracts algebraic varieties. Toric varieties. Fans.
(3) The orbit-cone correspondence. Properties of toric varieties. Smoothness.
(4) Projective toric varieties.
(5) Divisors and line bundles on toric varieties.
(6) Toric Surfaces.
(7) Toric Resolutions and Toric Singularities.
(8) Applications.
Period: February 16, 2015–April 30, 2015
30 hours

3. *Mathematical Analysis*

Title of the Course: **Partial Differential Equations** (LM Matematica Verona)
Lecturer: Marco Squassina

1. Harmonic functions: fundamental solution; mean value theorem; maximum principle; estimates on derivatives; Liouville theorem; analiticity; Harnack inequality
2. Properties of Sobolev spaces: finite increments, elliptic regularity
3. Elliptic equations: weak formulation; existence theorems via Fredholm alternative; weak maximum principle; Hopf lemma; strong maximum principle; inner regularity H^2
4. Parabolic equations: weak formulation; a priori estimates (Galerkin), esistence, uniqueness, regularity ad maximum principle
5. Elements of Calculus of Variations: first and second variation, convexity and weak semicontinuity, theorems on the exixtence of a minumum. Unicity results.
6. Elements of Non-linear Analysis: Mountain pass theorem and semi-linear elliptic equations. Over and under solutions. Non-esistence via Pohozaev

Period: February 16, 2015–April 30, 2015
30 hours

4. *Numerical Analysis*

Title of the Course: **Introduction to the Boundary Element Method**
Lecturer: Salim Meddahi, University of Oviedo (Spain)

The aim of this course is to introduce numerical technique that solves boundary value problems by means of boundary integral equations. The discretization of these boundary integral equations is commonly known as the boundary element method (BEM) and it is especially adapted for solving partial differential equations posed in unbounded domains. Although, in spirit, the BEM is essentially a finite element method for an integral equation, the analysis of the BEM is more challenging from the mathematical and numerical point of view. In this course, rather than attempting a general approach, we provide a comprehensive analysis of the method for the Laplace equation. Understanding the founding principles for this theory in this basic context paves the way for the application of the method to a wider class of partial differential equations. This is illustrated by extending the theory to the Helmholtz equation and by showing some applications from acoustic scattering theory. We also describe a strategy in which the BEM is applied in association with the finite element method in order to solve general elliptic boundary value problems with an exact representation of the radiation condition.

1 - Boundary Integral Equations for the Laplace Problem
2 - Boundary Integral Representations for the Helmholtz Equation
3 - Variational formulations for the coupling of boundary and finite element methods
4 - Boundary integral methods for the Laplace and the Helmholtz equations
5 - Boundary and finite element coupling methods

5. Mathematical Physics

Title of the Course: **Mathematical Physics** (LM Matematica Trento)
Lecturer: Enrico Pagani
Tensor calculus, differential geometry, fiber-bundles, connections, jet-spaces, Lie groups.
Analytical mechanics. Symplectic manifolds. Lagrangian and Hamiltonian formulation of Classical Mechanics. Non-holonomic constraints.
Geometric theory of first order partial differential equations. Wave propagation. Calculus of Variation in presence of non-holonomic constraints. Geometric Optimal Control Theory. Second variation, conjugate points, Maslov theory.Continuum Mechanics.
Ideal and viscous fluids. Elasticity theory.
Basic notions of Special and General Relativity Theory.
B. A. Dubrovin, S. P. Novikov, A. T. Fomenko, Geometria Contemporanea, Editori Riuniti/Mir, 1987
V. I. Arnold, Metodi Matematici della Meccanica Classica, Editori Riuniti, 1979
R. Abraham, J. Marsden, Foundations of Mechanics, Benjiamin Cummings, Reading, 1978
R. Abraham, J. Marsden, T. Ratiu, Manifolds, tensor analysis, and applications, Springer, 2003
R. Courant, D. Hilbert, Methods of Mathematical Physics, Interscience Publ., 1937
S. W. Hawking, G. F. R. Hellis, The large scale structure of space-time, Cambridge U.P., 1973
P. Olver, Equivalence, Invariants and Symmetries, Cambridge University Press, 1995
Period: February 16, 2015–April 30, 2015
30 hours

6. Probability Theory

Title of the Course: **Stochastic Differential Equations** (LM Matematica Trento)
Lecturer: Stefano Bonaccorsi

Program:
1) Gaussian measures in infinite dimensional spaces
2) Brownian motion, stochastic integral
3) Ito formula
4) soluzions of stochastic differential equations with additive and multiplicative noise
5) Applications and examples.
Period: February 16, 2015–April 30, 2015
30 hours

7. Operational research

Title of the Course: **Operations Research: mathematical algorithms for the decisions**
Lecturer: Romeo Rizzi (Verona)
Program:
This course is meant as a show-window of current lines of algorithmic research in discrete and applied mathematics. The aim is to present and propose recent lines of research on the crossroad between graph theory, algorithms, discrete mathematics, management of temporal constraints, combinatorial games, planning under uncertainty.
30 hours