Doctoral School in Environmental Engineering

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Free-discontinuities problems in signal and image segmentation

Abstract

Segmentation can be described as the process of partitioning a domain into disjoint and homogeneous regions according to some criterion while detecting the regions boundaries. Regions boundaries can be used to distinguish or recognize objects and shapes within the domain. In image analysis, segmentation is useful as a pre-classification process producing a smooth image, with preserved regions boundaries, which is easier to classify than the original image.

In signal analysis, segmentation is useful to identify signal discontinuities and to smooth the signal. The segmentation process can be seen as a free-discontinuities problem since the unknown is a pair (u,K) where u approximates the given signal/image and K is the discontinuities set of given signal/image. The discontinuities set K is both an unknown and a variable of the problem. Free-discontinuities problems can be approached in the calculus of variations framework.

With this perspective, the segmentation can be performed by mean of variational methods: the solution is derived by the minimization of some energy functionals expressed in terms of contours and interiors of the segmenting regions.

Direct methods in the calculus of variations can be applied to study the minimizer from the mathematical point of view.

From the numerical point of view difficulties arise in the detection of the unknown discontinuities. This leads to the definition of variational approximations employing different energy functionals. The equivalence between the original functional and the approximating functional is guaranteed by the use of the gamma-convergence technique.

The work mainly focus on the understanding of the above mathematical aspects and on their application to image and signal segmentation.

Original software has been developed and applied to synthetic and real images and one dimensional signals.

For the image segmentation, the Mumford and Shah functional has been studied and numerically implemented. The solution of the Mumford and Shah model is a piecewise

smoothed image with a set of abrupt discontinuities, the edges of the input image. Image smoothing and edge detection are intrinsically performed simultaneously. The segmentation is achieved by minimizing a functional composed by three terms. The first term forces the solution to approximates the given image, the second term forces the solution to be smooth within each segmenting region while it is allowed to vary along the discontinuity set of the given image and the third term asks the length of the discontinuity set to be minimal. Moreover, an extension of the original formulation add a term to control the curvature of the discontinuity set has also been studied and numerically implemented.
The MS model has been made available as a GRASS GIS module and has been tested on and applied to synthetic and remote sensed images. Results are in very good agreement with the well known properties of the model. Moreover, an unsupervised classification algorithm has been applied on an real image and on a segmented version of the same image in order to evaluate and underline the advantages of image segmentation.

For the one dimensional signal segmentation, the Blake and Zisserman functional has been studied and numerically implemented. The main difference with respect to the Mumford and Shah functional lies on the presence of a term controlling the signal second derivative whereas in the Mumford and Shah functional the first derivative is controlled. The solution of the Blake and Zisserman functional can therefore smoothly approximates ramps avoiding the so called "gradient effect".
The one dimension model of the Mumford and Shah functional has also been implemented.

The Blake and Zisserman functional has been applied to the segmentation of GPS permanent stations coordinates time series. In Geodesy and Geophysic, GPS coordinates time series are deeply studied. The segmentation of these time series aims to the detection of discontinuities on the station positions, such jumps can be due to crustal deformations, sensor anomalies or to human actions (e.g. sensor replacement). Also in the one dimensional case very good signals segmentations have been obtained.

Perspectives and future works, both from the mathematical point of view and from the environmental engineering point of view, are finally reported.