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Fluid Dynamics and Electromagnetism:

Theory and Numerical Approximation

(on the occasion of

Prof. Paolo Secchi and Prof. Alberto Valli 60th birthday)

Levico Terme (Trento, Italy) – June 3rd-6th, 2014

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BOOK OF ABSTRACTS / TALKS

TALKS

On regular boundary points for the $p-$Laplacian in N space variables

Hugo Beirão da Veiga

Università Pisa (Italy)

I turn back to some pioneering results concerning, in particular, nonlinear potential theory and the non-homogeneous boundary value problem $\mathcal{L}\,u = 0$ in Ω, $u = \phi$ on $\partial\Omega$, where $\mathcal{L}$ is the $p-$ Laplacian, $1 < p < N$, or a similar operator, and Ω is an $N-$dimensional, open bounded set. These results, obtained at the very beginning of the seventies, were kept in the shade.

To each arbitrary boundary data $\phi \in C(\partial\Omega)$ it corresponds a unique generalized solution $u \in C(\Omega)$, which satisfies the above boundary condition in a suitable sense. Further, a boundary point y is said to be regular if $u \in C(\Omega \cup \{y\})$ for all $\phi \in C(\partial\Omega)$. We have looked for really explicit geometrical conditions, sufficient to guarantee that a boundary point y is regular. We show that "little sharpened" $(N-1)$-dimensional external cusps, with vertex at the boundary point y, are sufficient to guarantee regularity. In particular, an $(N-1)-$dimensional external cone with vertex at y, is sufficient for regularity.

On the regularity of solutions of some boundary values problems arising in fluid mechanics

Luigi C. Berselli

Università Pisa (Italy)

Joint work with C.R. Grisanti.

We consider as particular example the scalar equations

$$-\mu\Delta u + (\delta + |\nabla u|)^{p-2}\nabla u = f,$$

in a smooth and bounded domain $\Omega \subset \mathbf{R}^n$, $n \geqslant 2$ and we focus on the regularity properties in the case $p < 2$. We supplement the system with vanishing Dirichlet data at the boundary, which is smooth but not necessarily flat.

By simplifying and adapting to case $p < 2$ techniques taken from [1, 2] we prove some new regularity results, with particular emphasis in the role of the constants $\mu \geqslant 0$ and $\delta \geqslant 0$, with possible applications to the p-Stokes system modeling slow motion of certain non-Newtonian fluids.

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Quadratic interaction potentials for conservation laws

Stefano Bianchini

SISSA Trieste (Italy)

We prove a quadratic interaction estimate for approximate solutions to conservation laws obtained by the wavefront tracking approximation or the Glimm scheme. This quadratic estimate has been used in the literature to prove the convergence rate of the Glimm scheme.

The proof is based on the introduction of a quadratic functional $\mathfrak{Q}(t)$, decreasing at every interaction, and such that its total variation in time is bounded.

Differently from other interaction potentials present in the literature, the form of this functional is the natural extension of the original Glimm functional, and coincides with it in the genuinely nonlinear case.

Steady state solution of periodically excited nonlinear eddy current problems

Oszkár Biró

Graz University of Technology (Austria)

In many cases the steady state periodic solution of eddy current problems with periodic excitation is needed only. If the problem is linear, it is straightforward to obtain this solution either in the frequency domain applying harmonic balance techniques or in the time domain using discrete Fourier transforms. For nonlinear problems, this cannot be done, since the harmonics are all coupled. Using a fixed-point method to solve the nonlinear equations, however, enables using harmonic balance or discrete Fourier transforms within each nonlinear iteration. The method is described in the presentation and examples of its application are given.

Finite element spaces for electromagnetism on general meshes of quadrilaterals and cubes

Daniele Boffi

Università Pavia (Italy)

It is well known that the approximation of differential problems arising from electromagnetism requires appropriate finite element spaces. A very powerful setting for the study of this topic is the Finite Element Exterior Calculus: a compatible approximation of the de Rham complex guarantees the numerical stability of the discretized problem. When considering finite elements

on general meshes of (possibly distorted) quadrilaterals or cubes, it is important to check that the approximation properties of the resulting spaces do not deteriorate. We discuss a general setting for the construction of finite element differential forms based on a tensor product structure; the approximation properties of the spaces are studied as the form degree varies. It turns out that sufficient conditions for a fixed order of approximation become stronger and stronger as the form degree gets higher. This is a joint work with D.N. Arnold and F. Bonizzoni.

H(div)- and H(curl)-conforming Virtual Element Spaces

Franco Brezzi

IUSS-Pavia and IMATI-CNR Pavia (Italy)

Joint work with L. Beirão da Veiga, L.D. Marini, and A. Russo.

We present a general overview on $\mathbf{H}(div)$- and $\mathbf{H}(\mathbf{curl})$-conforming Virtual Element spaces. As it happens for traditional Finite Elements (as Raviart-Thomas, BDM, BDFM, Nedelec, and others), these are the basic ingredients for the discretization of mixed formulations, as well as valuable instruments in a number of different other applications. On particular meshes (e.g. on triangulations without hanging nodes) we recover the classical Finite. But Virtual Element spaces can be defined and used on much more general (polygonal and polyhedral) meshes.

A dynamic domain decomposition for Hamilton-Jacobi-Bellman equations

Simone Cacace

Università di Roma "La Sapienza"(Italy)

In this talk we present a new algorithm for the solution of Hamilton-Jacobi-Bellman equations related to optimal control problems. The key idea is to divide the computational domain into subdomains which are shaped by the optimal dynamics of the underlying control problem. This can result in a rather complex geometrical subdivision, but it has the advantage that every subdomain is invariant with respect to the optimal dynamics, and then the solution can be computed independently in each subdomain. The features of this dynamics-dependent domain decomposition can be exploited to speed up the computation and for an efficient parallelization, since the classical transmission conditions at the boundaries of the subdomains can be avoided. It is interesting to note that, although the method has been designed for hyperbolic equations, we can extend some of its features to semilinear equations of second order. We present and discuss some preliminary results related to the advection-diffusion equation. Several examples in two and three dimensions illustrate the properties of this dynamic domain decomposition method.

Joint works with E. Cristiani, M. Falcone, A. Picarelli.

Subsurface flows in discrete fracture networks: uncertainty quantification

Claudio Canuto

Politecnico di Torino (Italy)

Discrete Fracture Network (DFN) models are widely used in the simulation of subsurface flows; they describe a geological reservoir as a system of many intersecting planar polygons representing the underground network of fractures. The mathematical description is based on Darcys law, supplemented by appropriate interface conditions at each intersection between two fractures. Efficient numerical discretizations allow for a totally independent meshing of each fracture. We consider stochastic versions of such models, in which stochasticity is inserted in two ways: i) in the transmissivity coefficients of the fractures, assuming the fracture network given; ii) in the generation of the network, considering as stochastic such parameters as the location of fractures, their orientation, their size. In the latter case, fracture intersections may appear or disappear while varing the stochastic parameters. An Uncertainty Quantification analysis is performed, based on a collocation approach over sparse grids in parameter spaces.

Path decomposition techniques in systems of hyperbolic conservation laws

Andrea Corli

Università Ferrara (Italy)

I shall present some recent results obtained in collaboration with F. Asakura about the global existence of weak solutions to systems of conservation laws. These results are obtained by a novel technique, which mixes the famous wave-front tracking with the decomposition of shock waves into paths. Each path encodes not only the previous interactions but also subdivides the strengths of the waves into linear and nonlinear ones. This simplifies the proof of the global existence of solutions and also allows the study of asymptotic behaviour of solutions.

A linearized model for compressible flow past a rotating obstacle

Reinhard Farwig

Technische Universität Darmstadt (Germany)

In this talk we present recent results jointly obtained with Milan Pokorný, Charles University Prague.

Consider the flow of a compressible Newtonian fluid around or past a rotating rigid obstacle in $\mathbb{R}^3$. After a coordinate transform to get a problem in a time-independent domain we assume the new system to be stationary, then linearize and use Fourier transform to prove the existence of a unique solution in L^q-spaces. However, in contrast to the incompressible case with multipliers based on the heat kernel the multiplier functions will have a much more complicated structure...

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R. Farwig, M. Pokorný: *A Linearized Model for Compressible Flow past a Rotating Obstacle: Analysis via Modified Bochner–Riesz Multipliers.* Technische Universität Darmstadt, FB Mathematik, Preprint (2014).

Korteweg fluids and related problems

EDUARD FEIREISL

Institute of Mathematics, Prague (Czech Republic)

We are concerned the Euler-Poisson-Korteweg system with a general Korteweg tensor including the capilarity and quantum fluids models as special cases. Existence and well posedness problems will be discussed. In particular, we show the weak-strong uniqueness principle as well as the existence of global-in-time weak solutions for rather general class of initial data.

Phase transitions and traveling waves in compressible fluids

HEINRICH FREISTUHLER

Universität Konstanz (Germany)

This talk studies families of isothermal Navier-Stokes-Allen-Cahn systems that are parameterized by temperature. It shows that under natural assumptions, the possibility of traveling waves corresponding to phase boundaries arises during a phase transition at a critical temperature. Below that temperature, besides pairs of "Maxwell" states connected by waves with zero net mass flux, there exist also interfaces across which particles undergo a phase transformation.

Finite elements for Immersed Boundary Method

LUCIA GASTALDI

Università Brescia (Italy)

The aim of this talk is to discuss the performances of finite elements in the space discretization of the Immersed Boundary Method (IBM). This formulation for fluid-structure interaction problems is based on the fact that the computational domain is the union of the regions occupied by fluid and solid. The Navier-Stokes equations are considered every where and the presence of the structure is taken into account by suitable additional forces. We present a fully variational approach of the IBM which is based on the introduction of a distributed Lagrange multiplier which takes into account the constraint that the solid has the same velocity as the fluid in the region where the solid is located. This new approach is unconditionally stable. Moreover the numerical solution enjoys better local mass conservation.

Interface control domain decomposition method for multiphysics

PAOLA GERVASIO

Università Brescia (Italy)

We present the Interface Control Domain Decomposition (ICDD) method to address heterogeneous and multiphysics problems by overlapping subdomain splitting. Interface controls are unknown functions used as Dirichlet boundary data on the interfaces of an overlapping decomposition designed for solving boundary value problems. The controls are computed through an optimal control problem with interface observation. The main advantage of applying this approach to heterogeneous problems is to avoid sharp interfaces which would require an in depth knowledge of the local physical behavior (interface conditions) of the specific problem. In this talk we consider the Navier-Stokes/Darcy problem modeling the filtration of incompressible fluids through porous media, as well as the coupling between advection-diffusion and pure advection problems.

Solidification and separation in saline water: the formation of brine channels

CLAUDIO GIORGI

Università Brescia (Italy)

Motivated by the formation of brine channels, a continuum model for salt separation and phase transition in saline water is presented. The mass density and the concentrations of salt and ice are the pertinent variables describing saline water. Hence the balance of mass is considered for the single constituents (salt, water, ice). To keep the model as simple as possible, the balance of momentum and energy are considered for the mixture as a whole. However, due to the internal structure of the mixture, an extra-energy flux is allowed to occur in addition to the heat flux. The constitutive equations involve the dependence on the temperature, the mass density of the mixture, the salt concentration and the ice concentration, in addition to the stretching tensor, and the gradient of temperature and concentrations. The balance of mass for the single constituents eventually result in the evolution equations for the concentrations. A whole set of constitutive equations compatible with thermodynamics are established. A free energy function is given which allows for capturing the main feature which occurs during the freezing of the salted water. That is, the salt entrapment in small regions (brine channels) where the cryoscopic effect forbid complete ice formation.

Continuous and Discrete Helmholtz-type Decompositions

RALF HIPTMAIR

ETH Zürich (Switzerland)

The term Helmholtz-type decomposition of $\mathbf{H}(\mathbf{curl}, \Omega)$ refers to stable splittings of the form

$$\mathbf{H}(\mathbf{curl}, \Omega) = (H^1(\Omega))^3 + \mathbf{grad}\, H^1(\Omega)\ .$$

First mentioned in a work by Birman and Solomyak [1], splittings of this type have quickly become a key tool in both the theoretical and numerical analysis of variational problems arising from Maxwell's equations. They proved instrumental in

- the investigation of extension theorems and trace spaces for $\mathbf{H}(\mathbf{curl}, \Omega)$ [2], which is the fundamental energy space for electromagnetic fields,
- the derivation of regularity and compactness results [3],
- the analysis of boundary integral formulations related to Maxwell's equations [6],
- the design of auxiliary space preconditioners for $\mathbf{H}(\mathbf{curl}, \Omega)$-elliptic variational boundary value problems [7],
- and the development of a multigrid convergence theory for edge elements [4, 5].

My presentation will start with a proof of the existence of Helmholtz-type decompositions. Then I am going to outline the ideas behing few of the applications mentioned above.

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The work of late-onset blind topologists as a key to understanding visualization issues in 3-D and in finding variational approaches to algorithms

P. Robert Kotiuga

Boston University (USA)

When compared to the 2-D case, the relative difficulty of computing and visualizing 3-D magnetic fields is a well-known, yet poorly understood, mystery. As observed by Gauss, Listing, Maxwell and Tait, topological aspects are also considerably more subtle. In 3-D, the linking and twisting of curves replaces intersections of curves on surfaces, the helicity of a vector field takes on a special topological significance, and cuts for magnetic scalar potentials give insights into the knotting of current paths.

In this paper I will focus on four late-onset blind mathematicians who are pioneers in 3-D topology. In each case I will tie their work to variational methods and to finite element algorithm development for 3-D electromagnetics. They are:

1) Joseph Plateau whose work on minimal surfaces gives insights into what cant be expected of area minimizing cuts. Other topological considerations arising in Plateaus problem give insights into mesh generation.

2) Lev Pontryagin proved the existence of Seifert surfaces in an intrinsically 3-D manner years before Herbert Seifert found a proof which exploits knot projections. Pontryagins argument generalized to what is now known as the Pontryagin-Thom construction. The Pontryagin-Thom construction can be considered the starting point for the harmonic map approach to computing cuts by means of the finite element method.

3) Bernard Morin provided the insights required to visualize the eversion of the sphere. A key insight of his is that of a half-way model. This in turn, is related to stationary points of the Willmore energy functional. These ideas are becoming increasingly important in computer graphics.

4) Emmanuel Giroux works at the interface of 3-D contact geometry and 4-D symplectic geometry. His work is intimately tied to the topological characterization of force-free magnetic fields and variational methods describing plasma equilibria. The finite element discretization of the associated helicity functional has a particularly elegant Whitney-form discretization.

I will conclude this exploration with some take-away messages involving considerations of cognitive science and technical communication.

Isogeometric collocation methods for some structural mechanics problems

Carlo Lovadina

Università di Pavia (Italy)

In the framework of Isogeometric Analysis (IGA) collocation methods have been proposed as an appealing high-order low-cost alternative to standard Galerkin approaches. We here present collocation IGA methods for the following problems:

- Straight beams and curved rods modeled by a Timoshenko-like theory. We show that locking-free IGA collocation methods can be designed, without the need of any compatibility condition for the involved discrete spaces: the convergence behavior is solely dictated by usual stability conditions for the collocation points, along with the approximation features of the used discrete spaces. A rigorous justification of such a result is available.

- Reissner-Mindlin plates. Although the stability and convergence analysis is not available for this case, numerical results indicate that also for plate bending problems the IGA collocation methods are a valid alternative to a more standard Galerkin approach.

This work has been developed in collaboration with F. Auricchio (University of Pavia), L. Beirao da Veiga (University of Milan), J. Kiendl and A. Reali (University of Pavia).

Quantum Hydrodynamics (QHD) and Magneto Quantum Hydrodynamics (MQHD)

Pierangelo Marcati

Università de L'Aquila (Italy)

Joint work with P. Antonelli.

We show how to construct weak solutions with finite energy initial data without any smallness and smoothness restriction both in the QHD and the MQHD case. We also show the hydrodynamics connected to the Complex Schroedinger (or Ginzburg Landau) approximation. Finally we sketch results analyzing the Kolmogorov power spectrum for the quantum turbulence.

Basic principles of mixed Virtual Element Methods

L. Donatella Marini

Università di Pavia and IMATI-CNR (Italy)

Joint work with F. Brezzi and Richard S. Falk.

We present, in the simplest possible way, the extension of the Virtual Element Method to the discretization of $H(div)$-conforming vector fields. As we shall see, the methods presented here can be seen as extensions of the so-called BDM family to deal with more general element geometries (such as polygons with an almost arbitrary geometry). For the sake of simplicity, we limit ourselves to the 2-dimensional case, with the aim of making the basic philosophy clear. However, we consider an arbitrary degree of accuracy k (the Virtual Element analogue of dealing with polynomials of arbitrary order in the Finite Element Framework). Preliminary numerical results are presented.

Finite element analysis for a pressure-stress formulation of a fluid-structure interaction spectral problem

Salim Meddahi

Universidad de Oviedo (Spain)

Joint work with David Mora and Rodolfo Rodríguez.

We present an analysis of an elastoacoustic vibration problem employing a dual-mixed formulation in the solid domain. The Cauchy stress tensor and the rotation are the primary variables in the elastic structure while the standard pressure formulation is considered in the acoustic fluid. The resulting mixed eigenvalue problem is approximated by a conforming Galerkin scheme based on the lowest order Lagrange and Arnold-Falk-Winther finite element subspaces in the fluid and solid domains, respectively. We show that the scheme provides a correct approximation of the spectrum and prove quasi-optimal error estimates. Finally, we report some numerical experiments.

Hyperbolic equations of Von Karman type

ALBERT J. MILANI

University of Wisconsin (USA)

Joint work with P. Cherrier (UPMC, Paris VI).

We investigate weak solutions of a hyperbolic system of equations of Von Karman type on the whole space $\mathbb{R}^{2m}, m \geqslant 2$. The system has the form

$$u_{tt} + \Delta^m u = N(f, u, ..., u) + N(\varphi, ..., \varphi, u), \tag{0.1}$$

$$\Delta^m f = -N(u, ..., u), \tag{0.2}$$

where the m-linear form N is defined by

$$N(u_1, ..., u_m) := \delta^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m} \nabla^{\beta_1}_{\alpha_1} u_1 \dots \nabla^{\beta_m}_{\alpha_m} u_m, \tag{0.3}$$

$\delta^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m}$ being the Kronecker tensor. Problem (0.1)+(0.2) is a generalization of the so-called von-Karman equations of thin plates in space dimension 2 (i.e., $m = 1$), in which case the non-linear operator $u \mapsto N(u, u)$ reduces to the Hessian determinant of the second derivatives of u. We establish the existence of a global weak solution, and the local well-posedness of semiweak solutions to the initial value problem corresponding to (0.1)+(0.2), in a suitable framework of Sobolev spaces $H^{m+k} \times H^k$, $0 \leqslant k \leqslant m$.

A posteriori estimates for the plane wave discontinuous Galerkin method for the Helmholtz equation

PETER MONK

University of Delaware, Newark (USA)

Several methods for approximating the solution of the Helmholtz equation have been proposed that use local solutions on small elements to build a wave based solver. To help in constructing a robust solver, we are interested in developing a posteriori error indicators for such methods. I shall briefly survey two methods: The Partition of Unity Finite Element Method and the Least Squares Method before concentrating on the Plane Wave Discontinuous Galerkin Method. I shall present discuss ongoing attempts to derive a posteriori error estimates for these methods.

Optimal control of free boundary problems with surface tension effects

Ricardo H. Nochetto

University of Maryland, College Park (USA)

We consider a PDE constrained optimization problem governed by a free boundary problem. The state system consists of coupling the Laplace equation in the bulk with a Young-Laplace equation on the free boundary to account for surface tension, as proposed by P. Saavedra and L. R. Scott. This amounts to solving a second order system both in the bulk and on the interface. Our analysis hinges on a convex constraint on the control which always enforce the state constraints. Using only first order regularity we show that the control-to-state operator is twice Frechet differentiable. We improve slightly the regularity of the state variables and exploit this to show existence of a control together with second order sufficient optimality conditions. We prove that the state and adjoint systems have the requisite regularity for the error analysis (strong solutions). We discretize the state, adjoint and control variables via piecewise linear finite elements and show optimal first order error estimates for all variables, including the control. We conclude with a more realistic model governed by the Stokes equations in the bulk and slip boundary conditions on the free boundary: we deal with minimal Sobolev regularity of the domain boundary. This work is joint with H. Antil and P. Sodre.

Operator-adapted finite element methods for time-harmonic wave propagation problems

Ilaria Perugia

University of Vienna (Austria)

Several finite element methods used in the numerical discretization of wave problems in frequency domain are based on incorporating a priori knowledge about the differential equation into the local approximation spaces. This can be done by using Trefftz-type basis functions, namely functions which belong to the kernel of the considered differential operator (e.g., plane or circular/spherical waves). The resulting methods feature enhanced convergence properties with respect to standard polynomial finite elements. In this talk, we focus on a family of Trefftz-discontinuous Galerkin (TDG) methods. For the Helmholtz problem, as well as for the time-harmonic Maxwell equation and the Navier equation, TDG methods are unconditionally well-posed and quasi-optimal a in a mesh-dependent energy-type norm. In this talk, we present an overview of TDG methods for time-harmonic wave propagation problems and recent results in their error analysis.

Source modeling of neural sources from measurements of the bio-electromagnetic field

Michele Piana

Università Genova (Italy)

The inverse problem of modeling neural sources from dynamic measurements of the associated electromagnetic field is ill-posed in the sense of Hadamard. Therefore some regularization technique must be applied in order to reduce the numerical instability of the solution. This talk will first describe how Bayesian approaches can naturally realize such regularization exploiting a priori information on the solution encoded in appropriate probability density functions. Then numerical implementations of these inversion methods will be discussed and their effectiveness will be shown in the case of experimental time series recorded by means of both magneto- and electroencephalography.

Model order reduction: basic concepts, basic applications

Alfio Quarteroni

Politecnico Milano (Italy) and EPFL Lausanne (Switzerland)

Model order reduction is a strategy to reduce the computational complexity in cases in which the latter would represent a major obstacle to the numerical solution of differential problems. Cases include parametrized partial differential equations, multi-query and real time problems. In this presentation I will recall the basic concepts behind the Reduced Basis method, its properties and the associated algorithmic aspects. Some simple applications of the method will be addressed, including those to optimal control of flow problems.

Spectral approximation of the curl operator in multiply connected domains

Rodolfo Rodríguez

Universidad de Concepción (Chile)

Joint work with Eduardo Lara and Pablo Venegas.
In a recent paper [1], two of the authors introduced and analyzed a couple of numerical methods based on Nédélec finite elements to solve the eigenvalue problem for the curl operator in simply connected domains. This topological assumption is not just a technicality, since the eigenvalue problem is ill-posed on multiply connected domains, in the sense that its spectrum is the whole complex plane, as is shown in [2]. However, additional constraints can be added to the eigenvalue problem in order to recover a well posed problem with a discrete spectrum [2, 3]. We choose as additional constraints a zero-flux condition of the curl on all the cutting surfaces. We introduce two weak formulations of the corresponding problem, which are convenient variations of those studied in [1]; one of them is mixed and the other a Maxwell-like formulation. We prove that both are well posed and show how to modify the finite element discretization from [1] to take

care of these additional constraints. We prove spectral convergence of both discretization as well as a priori error estimates. Finally, we report a numerical test which allows us to assess the performance of the proposed methods.

Key words: eigenvalue problems, topological constraints, finite element methods, spectral approximation.

Mathematics subject classifications (2010): Primary 65N15, 65N25, 65N30.

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Asymptotic behavior of solutions to Euler-Poisson equations for bipolar hydrodynamic model of semiconductors

Bruno Rubino

Univesità de L'Aquila (Italy)

We consider the Cauchy problem for the 1-D bipolar hydrodynamic model of semiconductor devices, the Euler-Poisson system with the physically relevant assumptions of non-flat doping profile and two different pressure functions:

$$\begin{cases} n_t + J_x = 0, \\ J_t + \Big(\dfrac{J^2}{n} + p(n)\Big)_x = nE - J, \\ h_t + K_x = 0, \\ K_t + \Big(\dfrac{K^2}{h} + q(h)\Big)_x = -hE - K, \\ E_x = n - h - D(x), \end{cases} \tag{0.4}$$

with the initial-value condition

$$\begin{cases} n(x,0) = n_0(x) \to n_\pm \text{ as } x \to \pm\infty, \\ h(x,0) = h_0(x) \to h_\pm \text{ as } x \to \pm\infty, \\ J(x,0) = J_0(x) \to J_\pm \text{ as } x \to \pm\infty, \\ K(x,0) = K_0(x) \to K_\pm \text{ as } x \to \pm\infty, \\ E(-\infty,t) = E_-. \end{cases} \tag{0.5}$$

Here, $n = n(x,t) > 0$, $h = h(x,t) > 0$, $J = J(x,t)$ and $K = K(x,t)$ represent the density of electrons, the density of holes, the current of electrons, and the current of holes, respectively, and

$E = E(x,t)$ is the electrical field. The nonlinear functions $p(n)$ and $q(h)$ denote the pressures of the electrons and the holes, respectively, which are usually different (more physical case) and satisfy:

$$p, q \in C^3(0, +\infty), \text{ with } s^2 p'(s) > 0 \text{ and } s^2 q'(s) > 0 \text{ strictly increasing for } s > 0. \tag{0.6}$$

$D(x) \neq 0$ is the doping profile standing for the density of impurities in semiconductor devices. $n_\pm$, $h_\pm$, $J_\pm$, $K_\pm$ and E_- are the state constants for the quantities at far fields.

This problem represents a physically relevant hydrodynamic model for a bipolar semiconductor device by considering two different pressure functions and a non-flat doping profile. Different from the previous studies (Gasser et al., 2003 [2], Huang et al., 2011 [3], Huang et al., 2012 [4]) for the case with two identical pressure functions and zero doping profile, in a recent paper by Donatelli et al., 2013 [1], we realize that the asymptotic profiles of this more physical model are their corresponding stationary waves (steady- state solutions) rather than the diffusion waves. Furthermore, we prove that, when the flow is fully subsonic, by means of a technical energy method with some new development, the smooth solutions of the system are unique, exist globally and time-algebraically converge to the corresponding stationary solutions. The optimal algebraic convergence rates are obtained.

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Transmission eigenvalues for a dielectric object resting on a perfect conductor

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Joint work with Peter Monk (Department of Mathematical Sciences, University of Delaware, Newark DE 19716, USA).

We introduce a new transmission eigenvalue problem with mixed boundary conditions that arises when a dielectric scatterer is mounted on a metal structure.

More precisely, we describe the associated forward problem and show that it has a unique solution by a reflection principle. We also formulate the inverse problem of identifying the shape and location of the scatterer from near field measurements. To solve numerically this inverse problem, we adapt the standard near field Linear Sampling Method (LSM). In particular, the analysis of the LSM involves a homogeneous interior transmission problem which has mixed boundary conditions.

In order to study this transmission eigenvalue problem, we rewrite it as a 4th order partial differential equation. This allows us to show that there exist infinitely many transmission eigenvalues and derive monotonicity, as well as a lower bound estimate for the first eigenvalue. We emphasize that our analysis mainly uses techniques from [4, 1, 2], and requires us to prove suitable density and compactness properties.

We finally provide numerical examples for the LSM and the approximation of transmission eigenvalues. These show that, for the examples that we have considered, both the shape of the scatterer and the mixed transmission eigenvalues can be approximated from near field data; cf. [3] for a study of the corresponding far field problem for standard transmission eigenvalues.

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A mathematical model of clot growth in flowing blood

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Blood coagulation is an extremely complex biological process in which blood forms clots (thrombus) to prevent bleeding; it is followed by their dissolution and the subsequent repair of the injured tissue. The process involves different interactions between the plasma, the vessel wall and platelets with a huge impact of the flowing blood on the thrombus growth regularization. A new mathematical model and some numerical results for thrombus development will be presented in this talk. The cascade of biochemical reactions interacting with the platelets, resulting in a fibrin-platelets clot production and the additional blood flow influence on thrombus development will be discussed. Two main aspects will be considered. The first one is the mathematical model reduction in terms of biochemical reactions to simplify the model complexity, allowing results in agreement with experimental data. The process will be initiated at the propagation phase, when the dominant part of thrombin and fibrin are produced, which requires an appropriate choice of the initial and boundary conditions to guarantee the prospective process development. A virtual equation to maintain the reliable prothrombinase production and additional platelets impact to the blood clot evolution is also included. The second feature of the model is to impose the slip velocity and the consequent supply of activated platelets in the clot region, showing its importance on the whole blood coagulation process. The model consists of a system of 13 nonlinear convection-reaction-diffusion equations, describing the cascade of biochemical reactions, coupled with a shear-thinning viscosity model for blood flow. Numerical results will be presented in a three-dimensional blood vessel, using the finite element method. Several cases of coagulation disorders leading to system perturbations will be also presented and compared with experimental results. The main objective of this study is to build a blood coagulation model able to predict effects of specific perturbations in the hemostatic system that can't be obtained by laboratory tests, and assist in clinical diagnosis and therapies of blood coagulation diseases.

This work has been done in collaboration with Antonio Fasano and Jevgenija Pavlova.

Stability of expanding flows by means of relative entropy estimates

Denis Serre

ENS Lyon (France)

We revisit a stability analysis induced by G.-Q. Chen, which uses a relative entropy method in order to prove that some compressible flows are asymptotically stable. By applying a global existence result of M. Grassin, we exhibit a very general class of stable flows, for which the flow map is expanding.

On some two phase problem – incompressible and compressible viscous flows separated by sharp interface

YOSHIHIRO SHIBATA

Tokyo, Waseda (Japan)

I would like to consider the evolution of incompressible and compressible viscous fluid flows separated by a sharp interface. I will talk about a local in time unique existence theorem in some general unbounded domain and a global in time unique existence theorem in a bounded domain. To prove local well-posedness the key is to show a maximal regularity theorem for the linearized equations, which is proved by using the $\mathcal{R}$-bounded solution operator to the corresponding resolvent problem with the help of Weis operator valued Fourier multiplier theorem. To prove global well-posedness, I use the continuation argument of time local solutions to arbitrary time interval. My argument is based on some decay theory for the linearized problem.

My talk is based on the following two papers:

[1] Takayuki Kubo, Yoshihiro Shibata and Kohei Soga, On the $\mathcal{R}$-boundedness for the two phase problem: Compressible-Incompressible Model Problem, submitted

[2] Takayuki Kubo and Yoshihiro Shibata, On the evolution of compressible and incompressible viscous fluids with a sharp interface, submitted

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On local existence of MHD contact discontinuities

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We study the free boundary problem for contact discontinuities in ideal compressible magnetohydrodynamics (MHD). They are characteristic discontinuities with no flow across the discontinuity for which the pressure, the magnetic field and the velocity are continuous whereas the density and the entropy may have a jump. We restrict ourselves to the case of 2D planar MHD flows, i.e., when the space variables $x = (x_1, x_2) \in \mathbb{R}^2$ and the velocity and the magnetic field have only two components: $v = (v_1, v_2) \in \mathbb{R}^2$, $H = (H_1, H_2) \in \mathbb{R}^2$. Under the Rayleigh-Taylor sign condition $[\partial p/\partial N] < 0$ on the jump of the normal derivative of the pressure satisfied at each point of the unperturbed contact discontinuity, we prove the well-posedness in Sobolev spaces of the linearized problem. This is a necessary step to prove the local-in-time existence of contact discontinuities in 2D planar MHD flows provided that the Rayleigh-Taylor sign condition is satisfied at each point of the initial discontinuity.

Joint work with: Alessandro Morando (*University of Brescia, Italy*) and Paola Trebeschi (*University of Brescia, Italy*)

The stability of stationary solutions to the MHD equations under the inhomogeneous boundary condition

Taku Yanagisawa

Nara Women's University (Japan)

We consider stationary weak solutions of the MHD equations in a bounded domain in $\mathbb{R}^3$ under the inhomogeneous boundary condition. We discuss the stability of the stationary solutions in L^2-framework.
