Topological features in photonics

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Fai della Paganella
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- Sunil Mittal, Vikram Orre, M. Khan, H. Kim, B. Cao
- S. Ganeshan, J. Fan, P. Adhikari, J. Taylor, A. Migdall (NIST/JQI)
- Vaezi (Cornell)
- M. Lukin, E. Demler (Harvard)
- P. Rabl (Vienna), C. Galland (Stuttgart)
- N. Harris, D. Englund (MIT)
- E. Kapit (CUNY), S. Simon (Oxford)
- F. Grusdt, F. Letscher, M. Fleischhauer (Kaiserslautern)

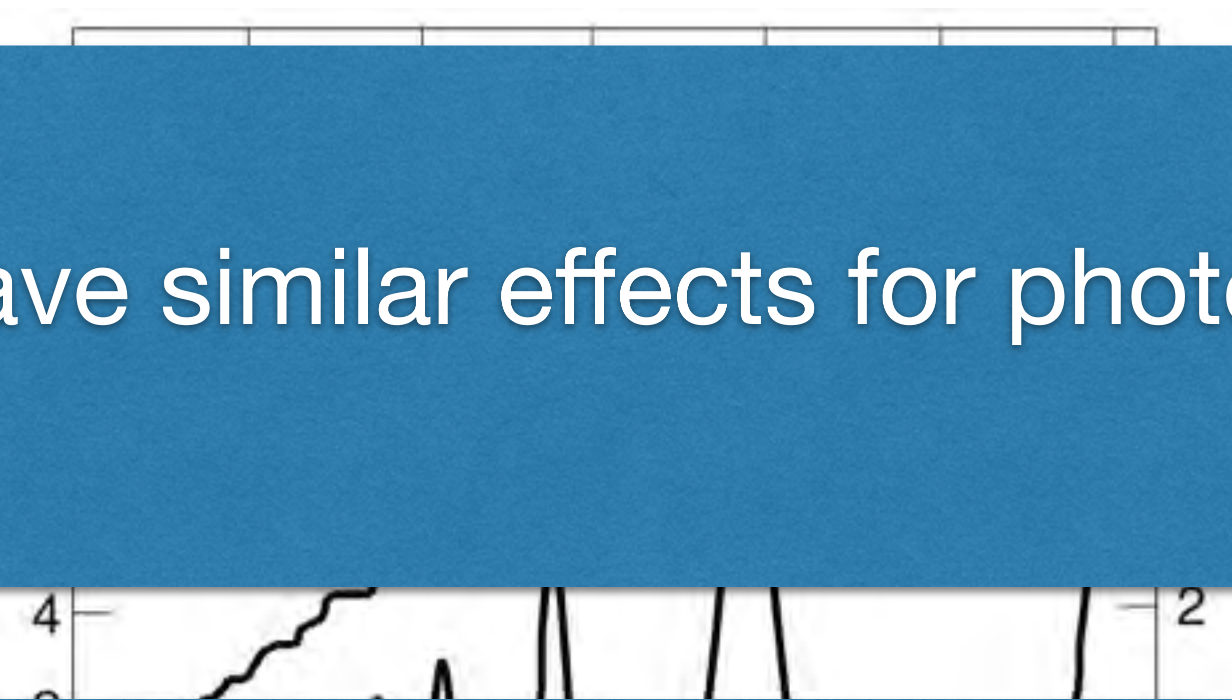
Physics Frontier Center

Joint Quantum Institute

publications: hafezi.umd.edu

- Synthetic gauge field, formalism
- Experiment in silicon photonics
- Interacting regime: Fractional quantum Hall
    - Laughlin, Pfaffian etc.
    - Preparation
- Measuring topological invariants

Review: Integer Quantum Hall effect



Is it possible to have similar effects for photons?

Microwave : Haldane, Raghu (2008), Soljacic (2009)

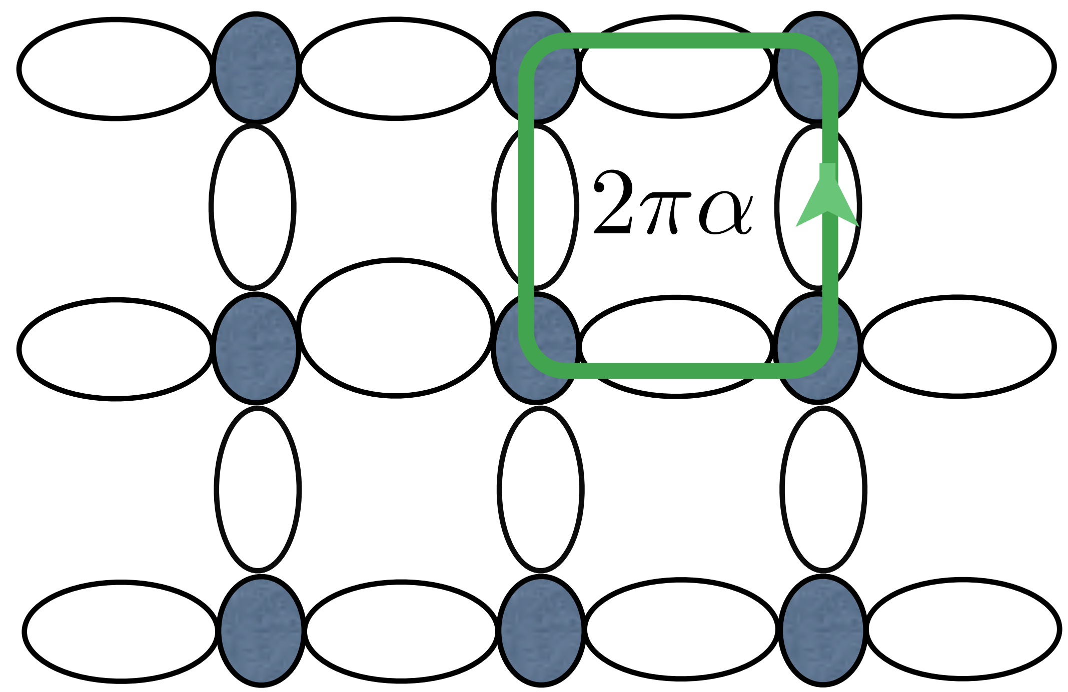
Optical domain: weak magneto-optic effect!

Standard for electrical conduction

Synthetic Magnetic Field

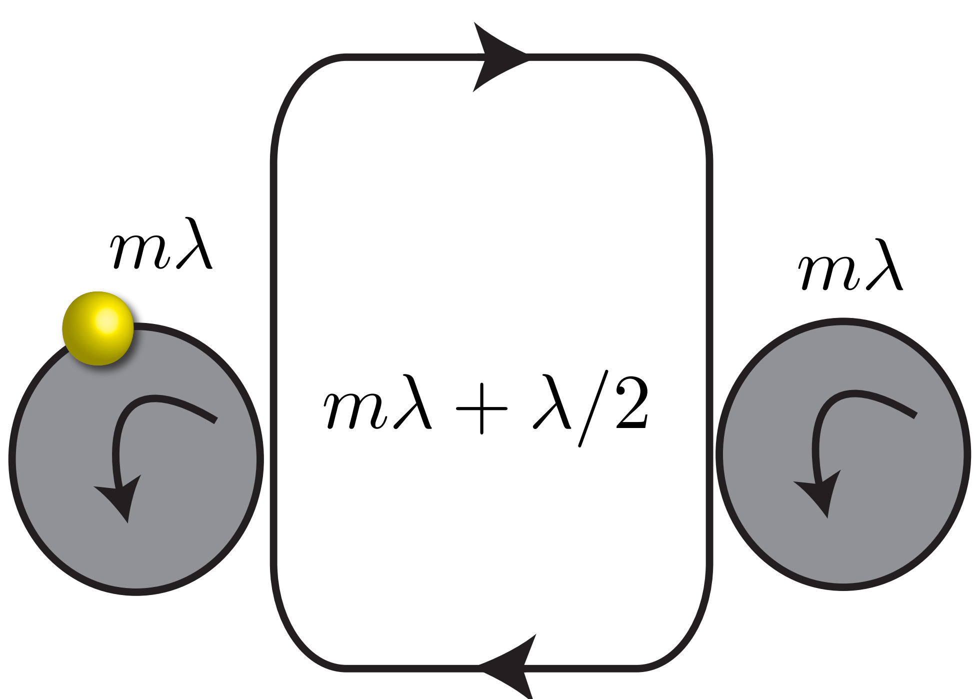
In analogy to electrons on a magnetic lattice:

$$
\begin{aligned}
H_0 &= -J\sum_{x,y}\hat{a}^{\dagger}_{x+1,y}\hat{a}_{x,y}e^{-i2\pi\alpha y}+\hat{a}^{\dagger}_{x,y}\hat{a}_{x+1,y}e^{i2\pi\alpha y}\\
&+ \hat{a}^{\dagger}_{x,y+1}\hat{a}_{x,y}+\hat{a}^{\dagger}_{x,y}\hat{a}_{x,y+1}
\end{aligned}
$$



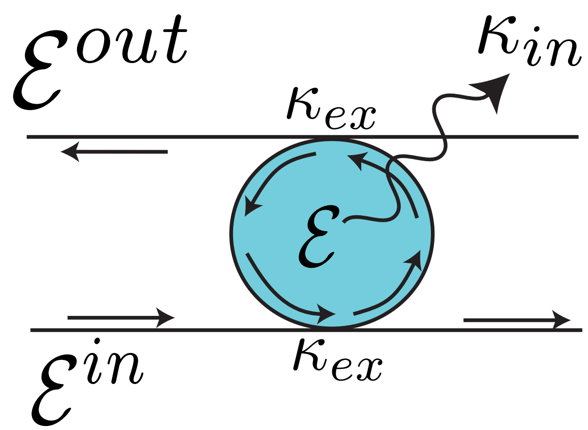
- Tight-binding form
- Magnetic phase

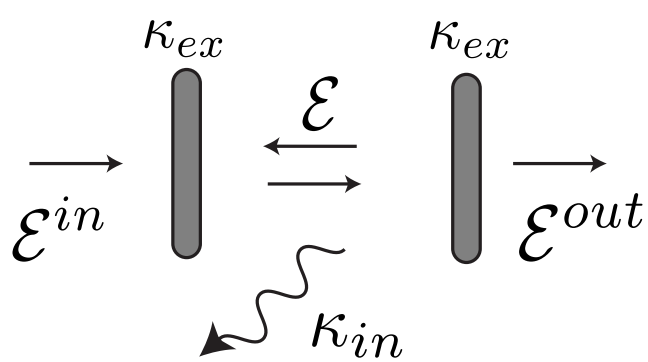
Two resonator case:



$$H_{eff} = -\kappa\hat{a}^{\dagger}_{x+1}\hat{a}_x e^{-2\pi i\alpha} - \kappa\hat{a}^{\dagger}_x\hat{a}_{x+1}e^{2\pi i\alpha}$$

Single-mode resonators: input-output formalism



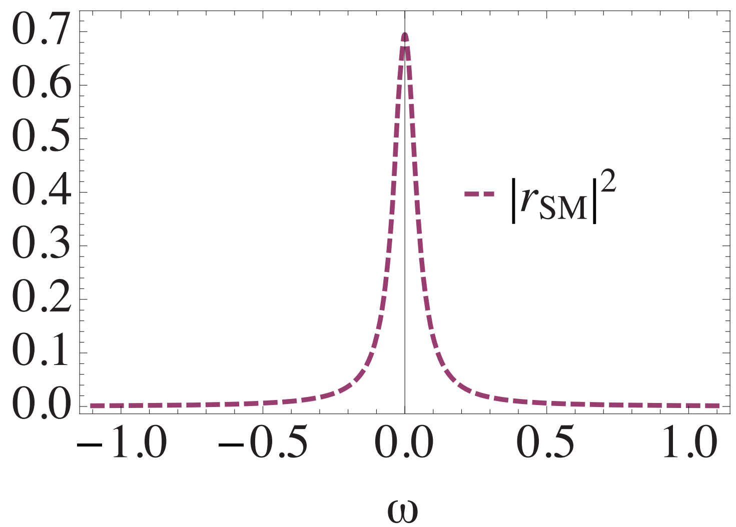


$$\frac{d\mathcal{E}}{dt} = (-\kappa_{in} - 2\kappa_{ex})\mathcal{E}$$

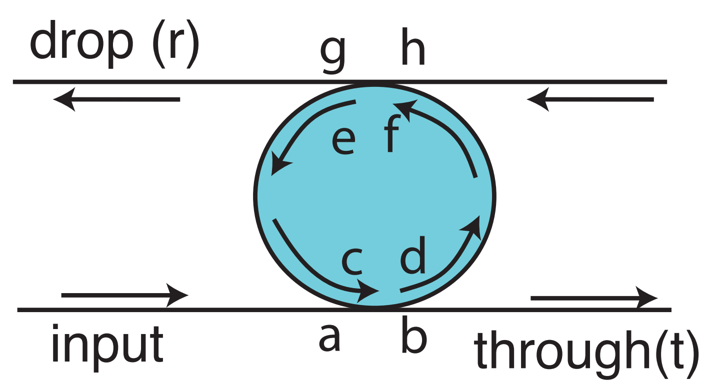
ω is the detuning

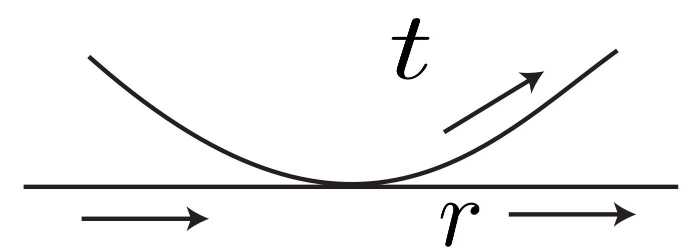
$$\mathcal{E}^{out} = \sqrt{2\kappa_{ex}}\mathcal{E}$$

$$r_{SM} = \frac{\sqrt{2\kappa_{ex}}\mathcal{E}}{\mathcal{E}_{in}e^{-i\omega t}} = \frac{2\kappa_{ex}}{i\omega - 2\kappa_{ex} - \kappa_{in}}$$



Multi-mode regime:
transfer matrix formalism





$$M_{coupl} = \frac{1}{t}\begin{pmatrix} -r^2+t^2 & r \\ -r & 1 \end{pmatrix}, \quad \begin{pmatrix} d \\ c \end{pmatrix} = M_{coupl}\begin{pmatrix} a \\ b \end{pmatrix}, \quad \begin{pmatrix} g \\ h \end{pmatrix} = M_{coupl}\begin{pmatrix} f \\ e \end{pmatrix}$$

$$|t|^2+|r|^2=1$$

$$M_{prop} = \begin{pmatrix} e^{i\beta \mathrm{L}/2-\alpha' \mathrm{L}/2} & 0 \\ 0 & e^{-i\beta \mathrm{L}/2+\alpha' L/2} \end{pmatrix}, \quad \begin{pmatrix} f \\ e \end{pmatrix} = M_{prop}\begin{pmatrix} d \\ c \end{pmatrix}$$

L is the total length of the resonator.

wave number: $\beta = n\omega/c$

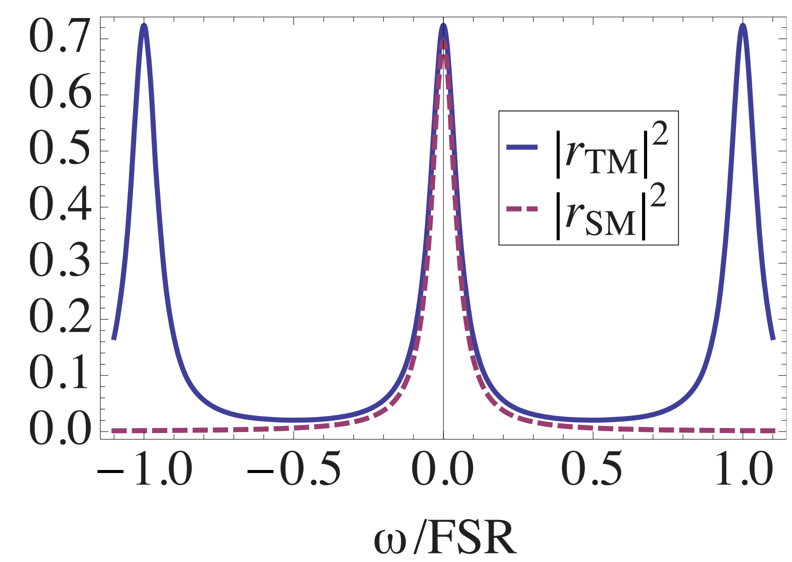
α' : propagation loss

Given a=1 and h=0, we can find (b, g)

Assume the coupling is weak: $r \to \sqrt{1-\epsilon^2}, t \to i\epsilon$

$$\epsilon \ll 1$$

$$FSR = 2\pi c/L$$



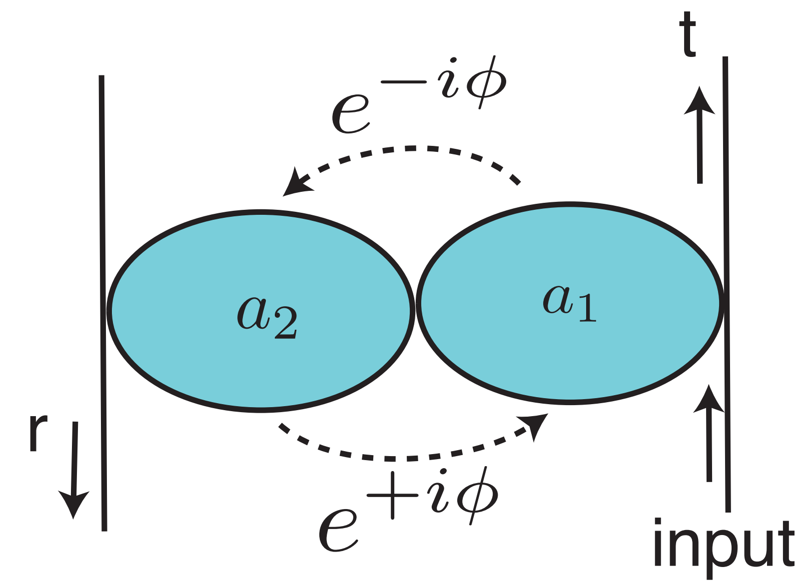
$$r_{TM} = \frac{\epsilon^2}{i\beta L - \alpha' L - \epsilon^2}$$

$$r_{SM} = \frac{\sqrt{2\kappa_{ex}}\mathcal{E}}{\mathcal{E}_{in} e^{-i\omega t}} = \frac{2\kappa_{ex}}{i\omega - 2\kappa_{ex} - \kappa_{in}}$$

$$\epsilon^2 \to \frac{4\pi\kappa_{ex}}{\mathrm{FSR}} \;,\quad \alpha'\mathrm{L} \to \frac{2\pi\kappa_{in}}{\mathrm{FSR}} \quad,\quad \beta\mathrm{L} \to 2\pi\frac{\omega}{\mathrm{FSR}}$$

Desired Hamiltonian, without coupling waveguides:

$$H = -J\hat{a}_2^\dagger \hat{a}_1 e^{-i\phi} - J\hat{a}_1^\dagger \hat{a}_2 e^{+i\phi}$$

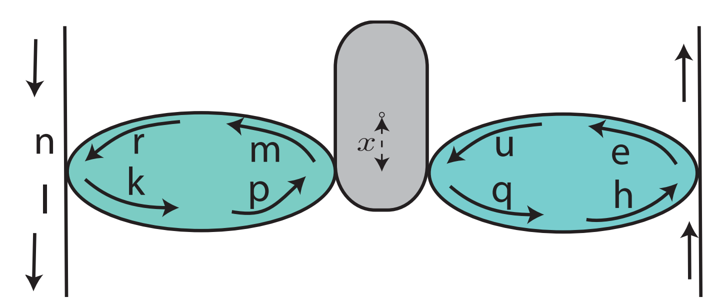


With coupling waveguides:

$$\frac{d}{dt}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -\kappa_{in} - \kappa_{ex} & iJe^{+I\phi} \\ iJe^{-I\phi} & -\kappa_{in} - \kappa_{ex} \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} - \sqrt{2\kappa_{ex}}\begin{pmatrix} \mathcal{E}_{in} \\ 0 \end{pmatrix}$$

$$a_2^{out} = \sqrt{2\kappa_{ex}} a_2, a_1^{out} = \mathcal{E}_{in} + \sqrt{2\kappa_{ex}} a_1$$

$$r_{SM} = \frac{\sqrt{2\kappa_{ex}} a_2}{\mathcal{E}_{in}} = -\frac{2ie^{-i\phi} J\kappa_{ex}}{J^2 + (i\omega - \kappa_{ex} - \kappa_{in})^2}$$

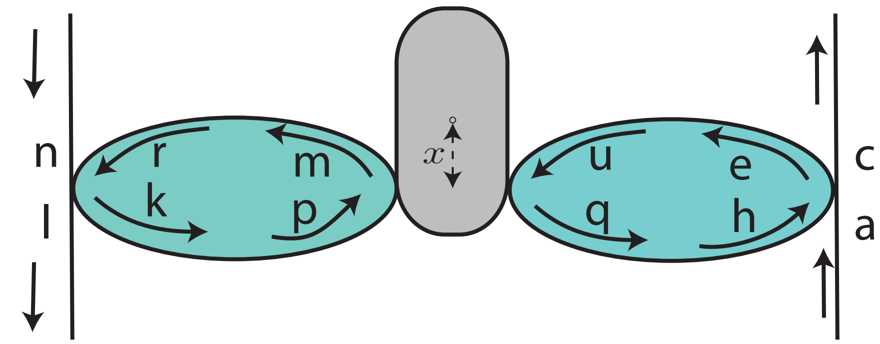


From transfer matrix approach: $r_{TM} = \dfrac{2e^{-i\phi} J\kappa_{ex}}{2J(\kappa_{ex} + \kappa_{in} - i\omega)\cos(\beta\eta) - i\left(J^2 + (\kappa_{ex} + \kappa_{in} - i\omega)^2\right)\sin(\beta\eta)}$

$$\epsilon_1{}^2 \to \frac{4\pi\kappa_{ex}}{\mathrm{FSR}}\ ,\quad \epsilon_2{}^2 \to \frac{4\pi J}{\mathrm{FSR}}\ ,\quad \alpha'\mathrm{L} \to \frac{2\pi\kappa_{in}}{\mathrm{FSR}}\ ,\quad \beta\mathrm{L} \to 2\pi\frac{\omega}{\mathrm{FSR}}\ ,\quad \beta\mathrm{x} \to \phi$$

$$r_{SM} = \frac{\sqrt{2\kappa_{ex}}a_2}{\mathcal{E}_{in}} = -\frac{2ie^{-i\phi}J\kappa_{ex}}{J^2 + (i\omega - \kappa_{ex} - \kappa_{in})^2}$$

$$r_{TM} = \frac{2e^{-i\phi}J\kappa_{ex}}{2J(\kappa_{ex} + \kappa_{in} - i\omega)\cos(\beta\eta) - i\left(J^2 + (\kappa_{ex} + \kappa_{in} - i\omega)^2\right)\sin(\beta\eta)}$$



For: $\beta\eta = \pi/2, 3\pi/2, ...$ both expressions become equal

$$H = -J\hat{a}_2^\dagger \hat{a}_1 e^{-i\phi} - J\hat{a}_1^\dagger \hat{a}_2 e^{+i\phi}$$

In general, $J_{eff} \rightarrow J/\sin(\beta\eta)$

$$\omega \rightarrow \omega - J\cot(\beta\eta)$$

Coupled Resonator Optical Waveguides (CROW)

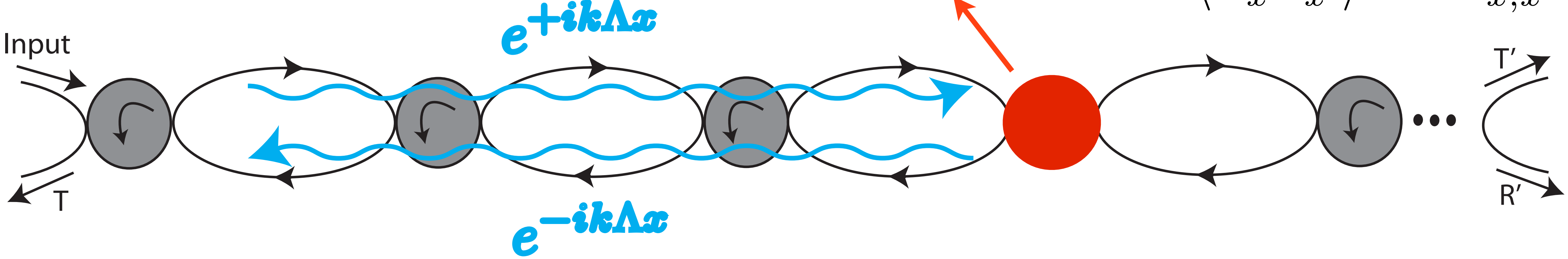
Tight-binding form:

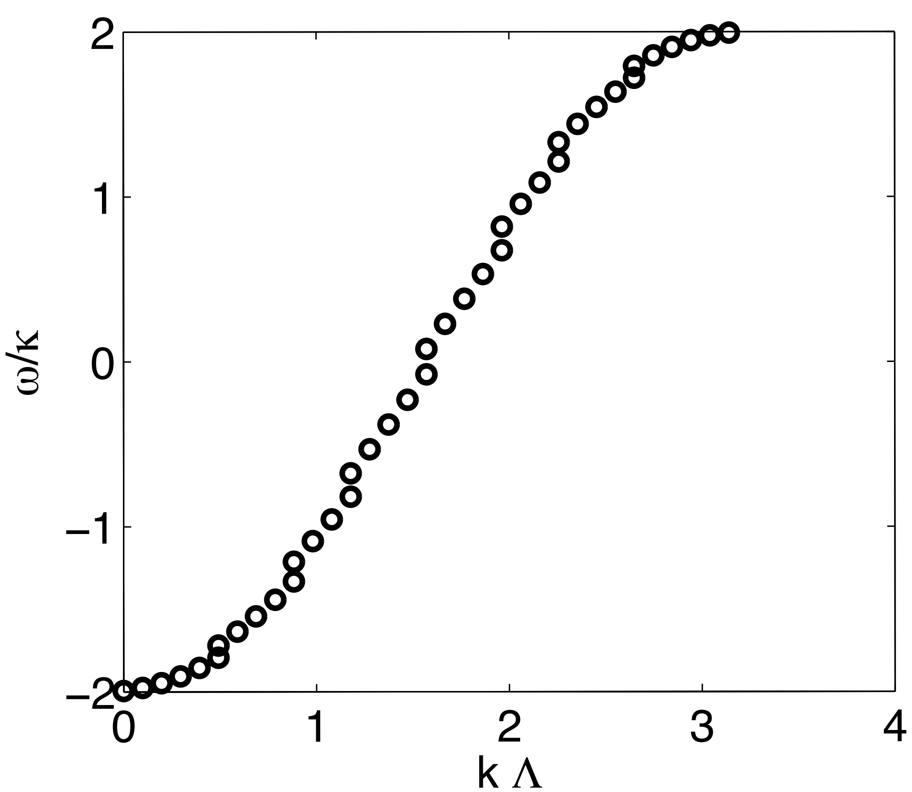
$$H_{eff} = -\kappa \hat{a}^\dagger_{x+1}\hat{a}_x - \kappa \hat{a}^\dagger_x \hat{a}_{x+1} + U_x \hat{a}^\dagger_x \hat{a}_x$$

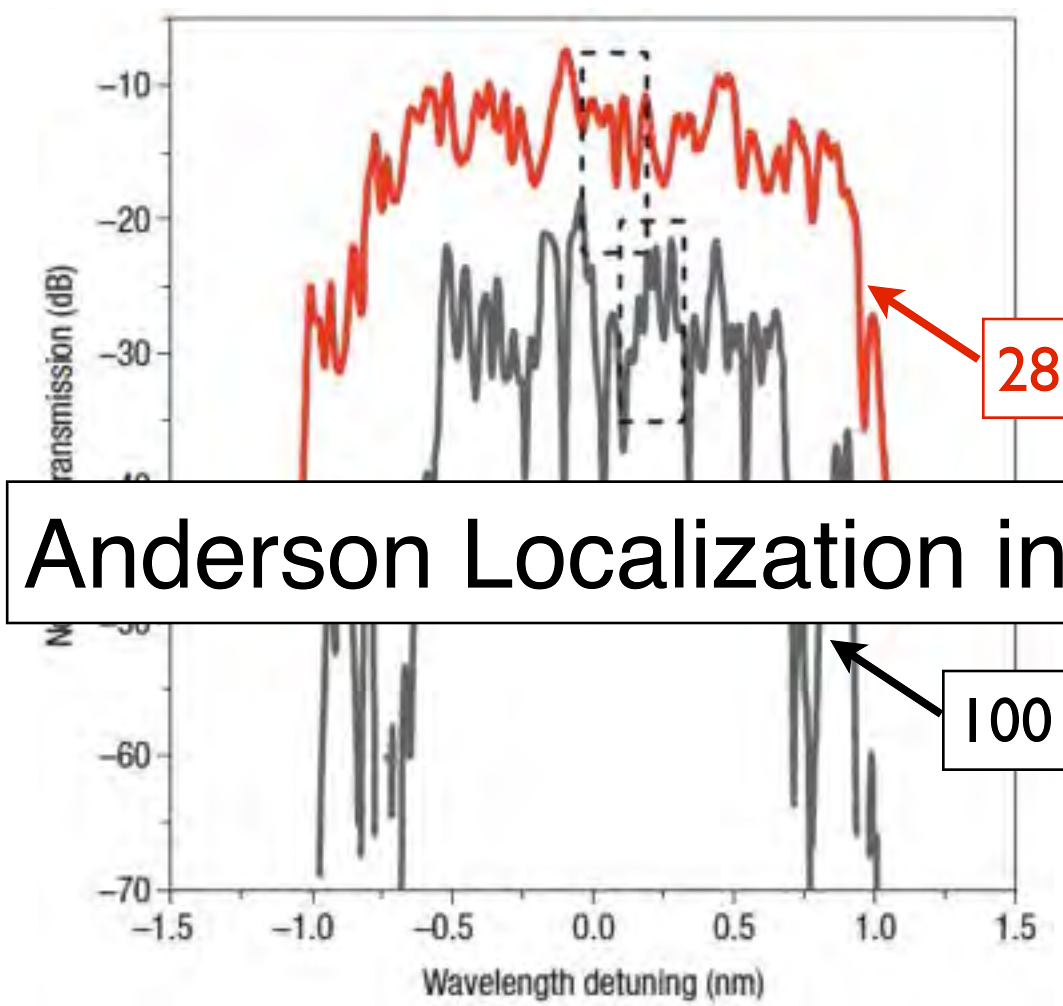
κ: resonator decay rate into the waveguide

$\langle U_x \rangle = 0$

$\langle U_x U_{x'} \rangle = U \delta_{x,x'}$







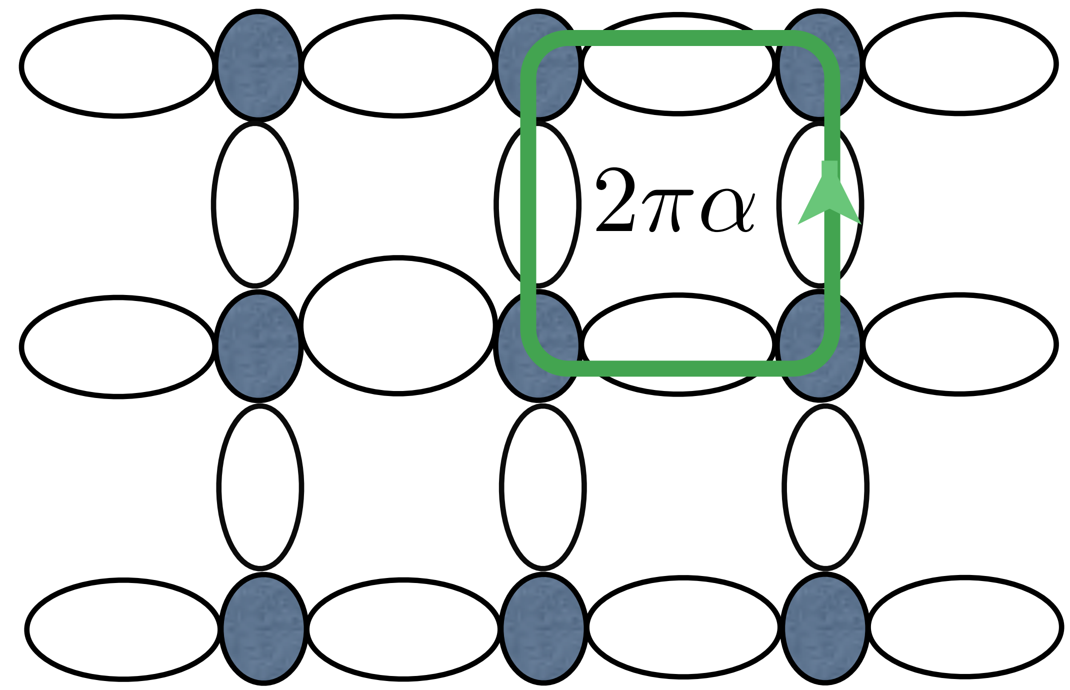
Anderson Localization in 1D

IBM, Nature Photonics (2007)

Synthetic Magnetic Field

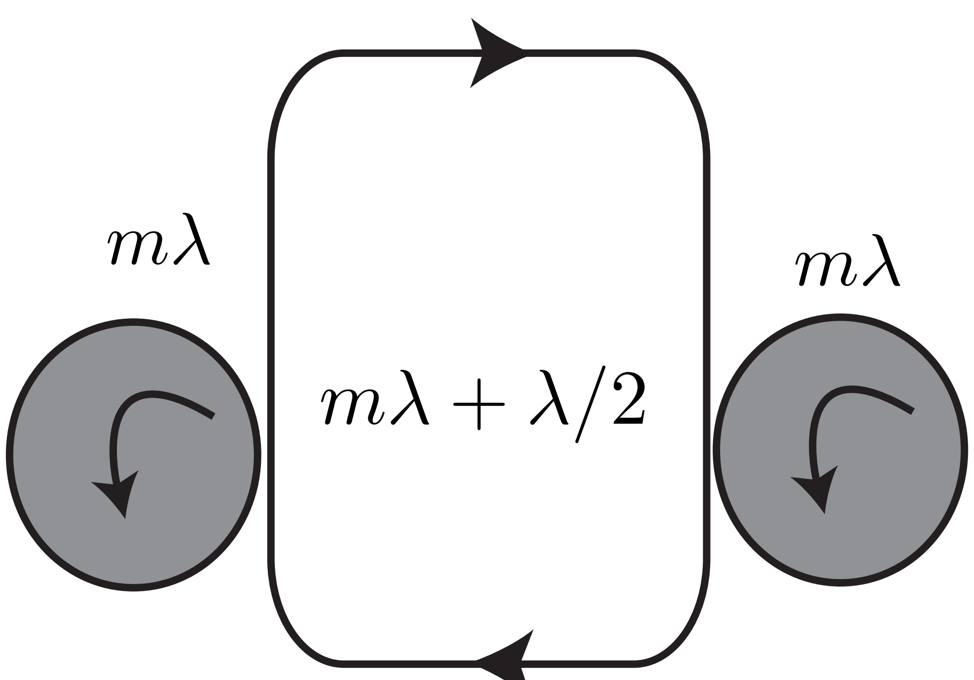
In analogy to electrons on a magnetic lattice:

$$
\begin{aligned}
H_0 &= -J\sum_{x,y} \hat{a}^\dagger_{x+1,y}\hat{a}_{x,y}e^{-i2\pi\alpha y} + \hat{a}^\dagger_{x,y}\hat{a}_{x+1,y}e^{i2\pi\alpha y} \\
&+ \hat{a}^\dagger_{x,y+1}\hat{a}_{x,y} + \hat{a}^\dagger_{x,y}\hat{a}_{x,y+1}
\end{aligned}
$$



- Tight-binding form
- Magnetic phase

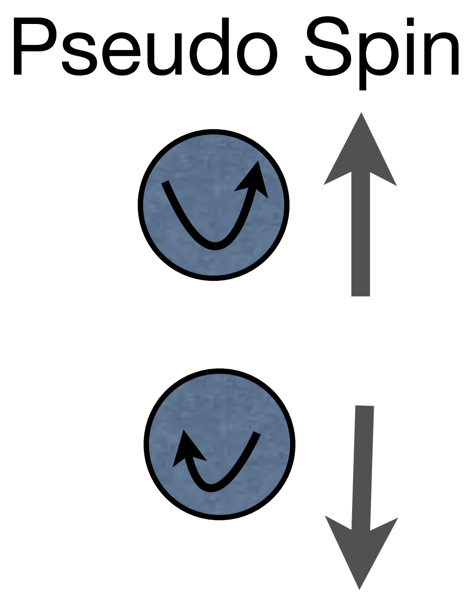
Two resonator case:



$$H_{eff} = -\kappa\hat{a}^\dagger_{x+1}\hat{a}_x e^{-2\pi i\alpha} - \kappa\hat{a}^\dagger_x\hat{a}_{x+1}e^{2\pi i\alpha}$$

MH, Demler, Lukin, Taylor Nat. Phys. 7, 907 (2011), see also Umucalilar and Carusotto PRA (2011)

Quantum Spin Hall



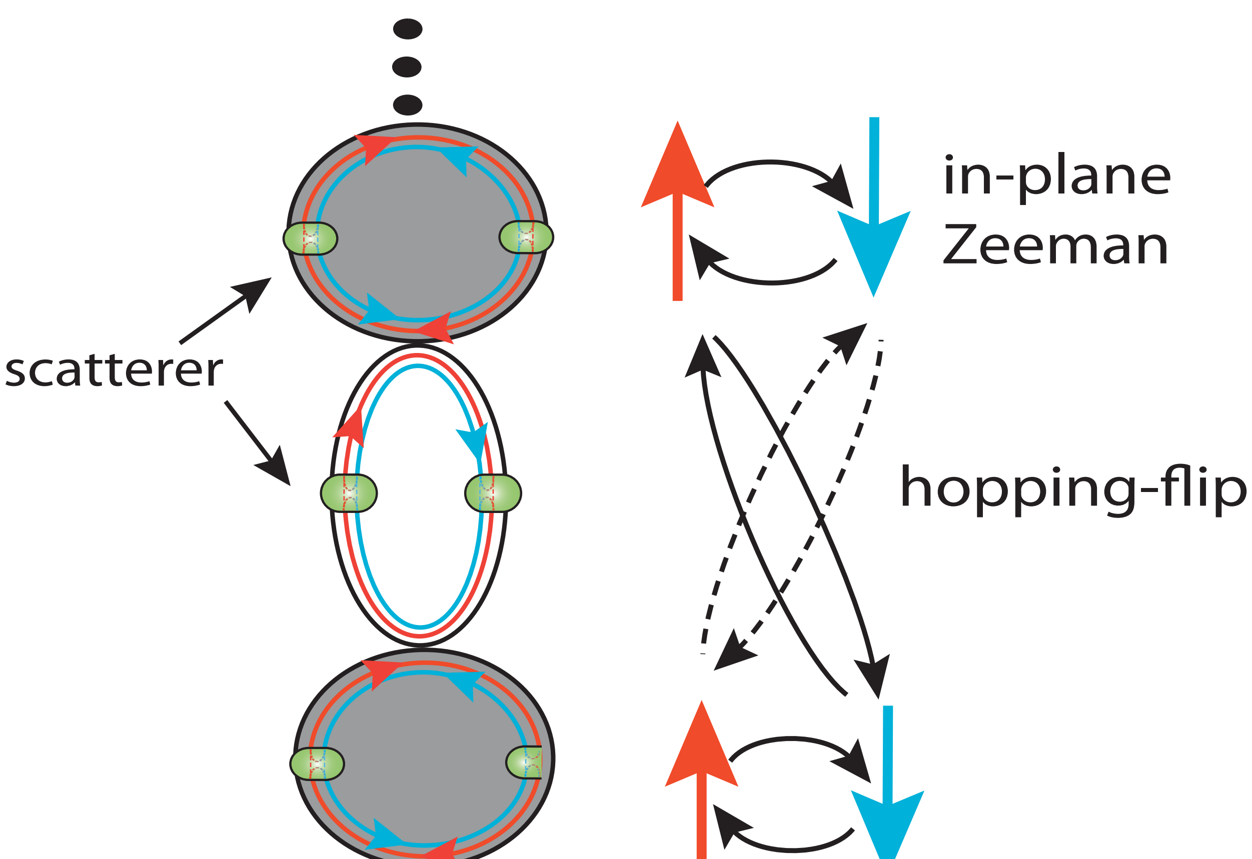
Magnetic field

$$+2\pi\alpha$$

$$-2\pi\alpha$$

$H_{SO} \rightarrow S.L$ time-reversal invariance

Adding spin-flip terms:

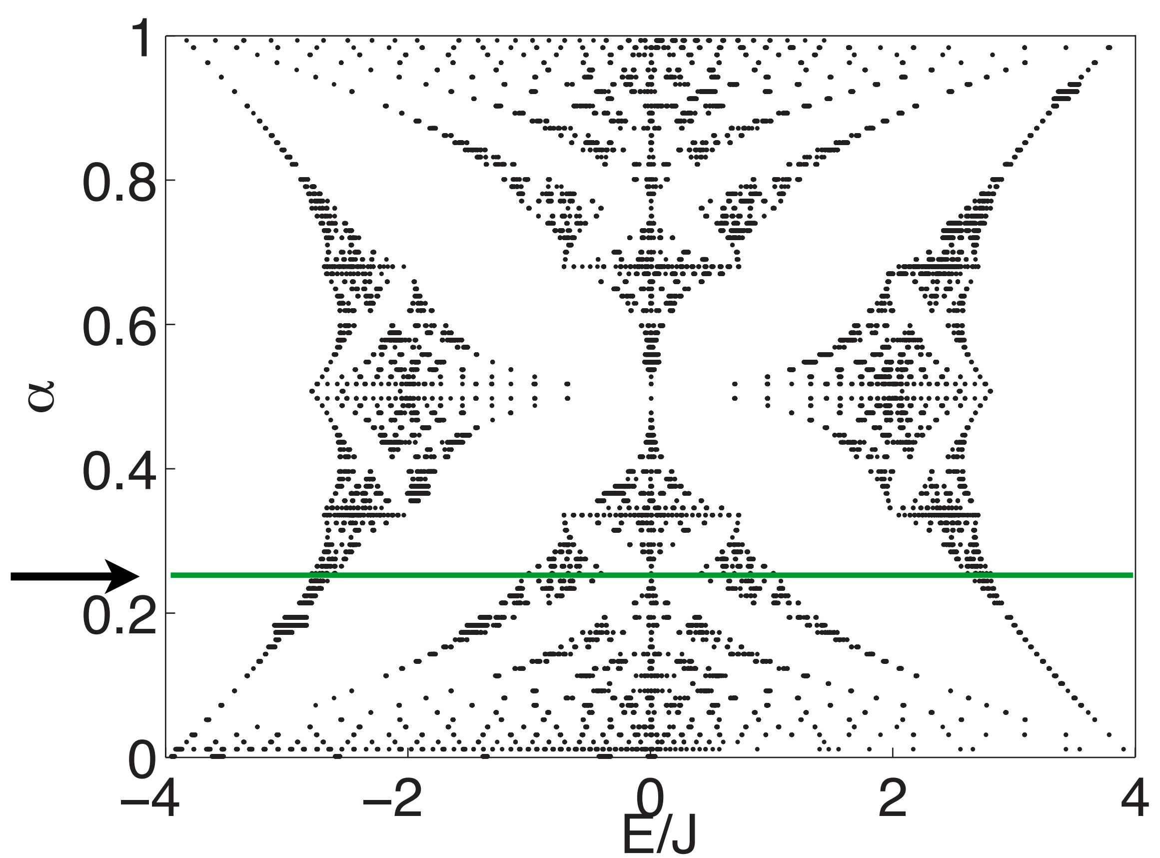


$$\begin{aligned} H &= -\sum_{x,,y} \left(a^{\dagger}_{\uparrow x,y+1} \quad a^{\dagger}_{\downarrow x,y+1} \right) \begin{pmatrix} \kappa & 0 \\ 0 & \kappa \end{pmatrix} \begin{pmatrix} a_{\uparrow x,y} \\ a_{\downarrow x,y} \end{pmatrix} + h.c. \\ &+ \frac{2}{\pi} \sum_{x,y} \left(a^{\dagger}_{\uparrow x,y} \quad a^{\dagger}_{\downarrow x,y} \right) \begin{pmatrix} 0 & \epsilon\kappa\mathcal{F} \\ \epsilon\kappa\mathcal{F} & 0 \end{pmatrix} \begin{pmatrix} a_{\uparrow x,y} \\ a_{\downarrow x,y} \end{pmatrix} \end{aligned}$$

$$H = -\sum_{x,y} \left(a^{\dagger}_{\uparrow x,y+1} \quad a^{\dagger}_{\downarrow x,y+1} \right) \begin{pmatrix} \kappa & \kappa\epsilon \\ \kappa\epsilon & \kappa \end{pmatrix} \begin{pmatrix} a_{\uparrow x,y} \\ a_{\downarrow x,y} \end{pmatrix} + h.c.$$

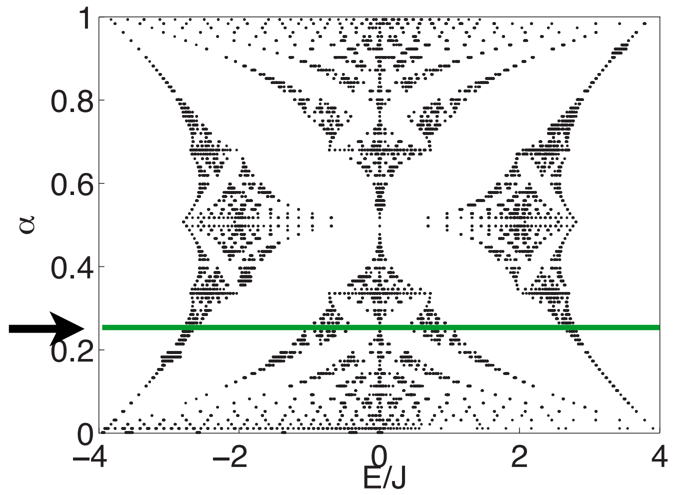
Probing photonic system

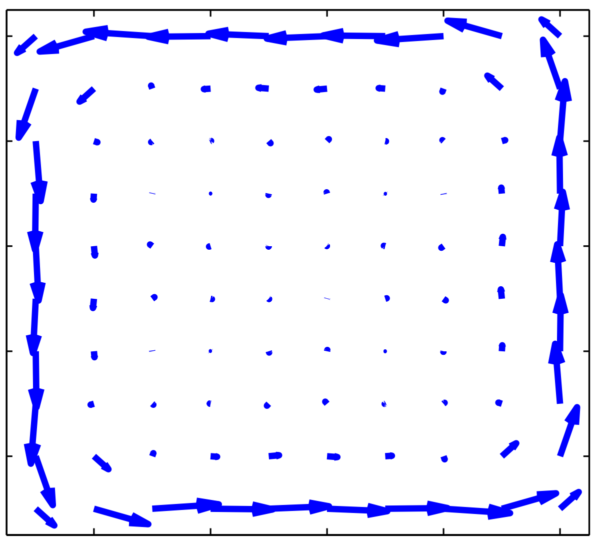
- Probing allows us to perform a “spectroscopy” of photonic states
- Energy spectrum of an infinite tight-binding with uniform magnetic field is Hofstadter butterfly
- Torus boundary condition (10x10) can simulate an infinite system

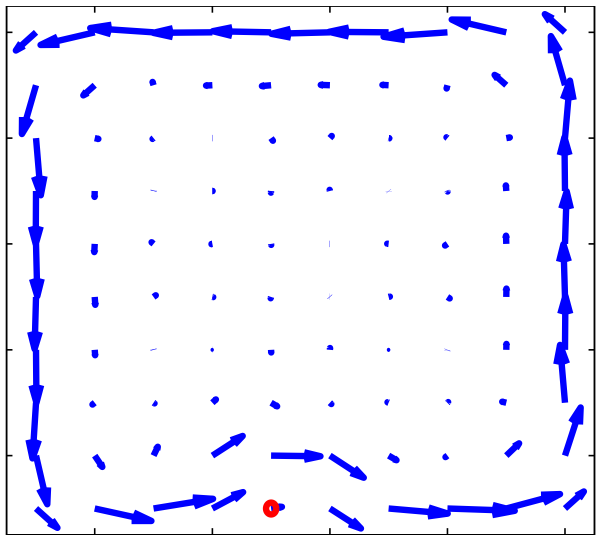


Edge states

- Finite system has edges -> New states beside Hofstadter states
- Quasi-one dimensional states that are localized at the perimeter of the system
- Carry a chiral current





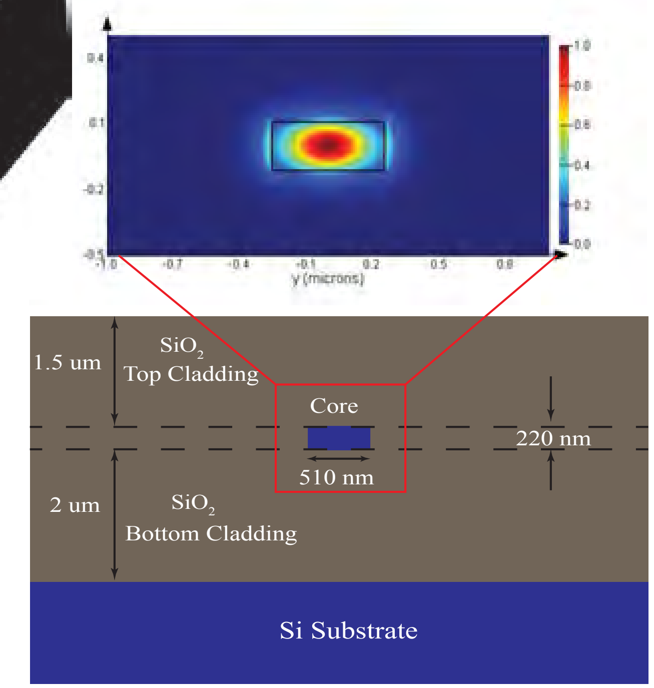


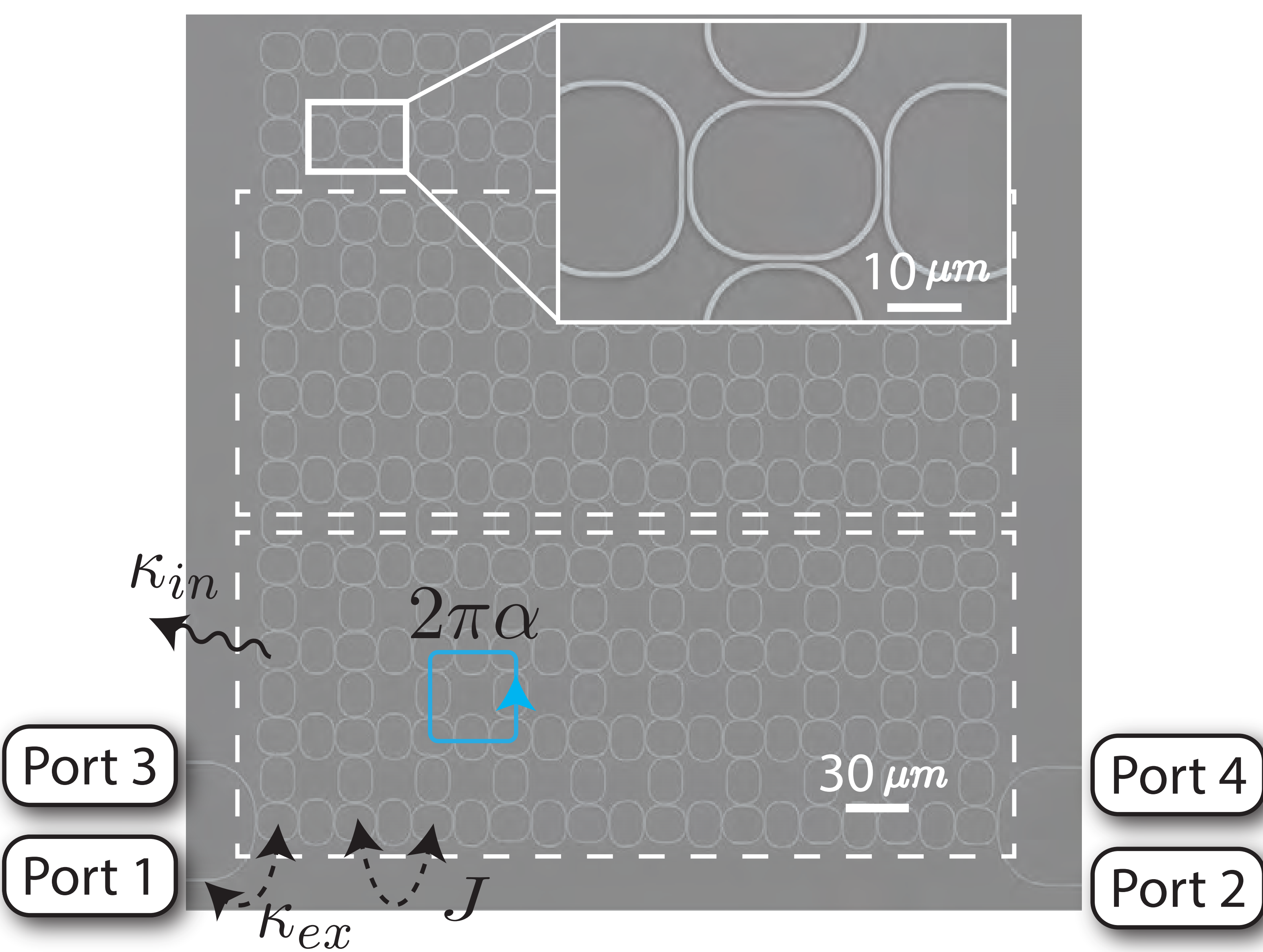
- Robust against non-magnetic impurities $U a_i^\dagger a_i$
- Current "gets around" the impurity rather than getting scattered in the opposite direction

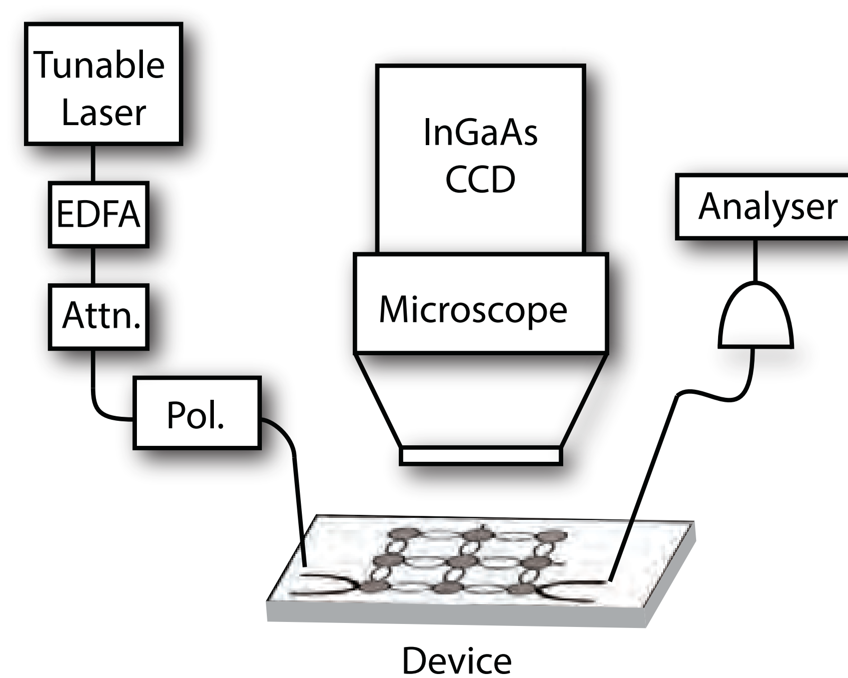
- Synthetic gauge field, formalism
- Experiment in silicon photonics
- Interacting regime: Fractional quantum Hall
  - Laughlin, Pfaffian etc.
  - Preparation
- Measuring topological invariants

Experimental realization of the gauge field

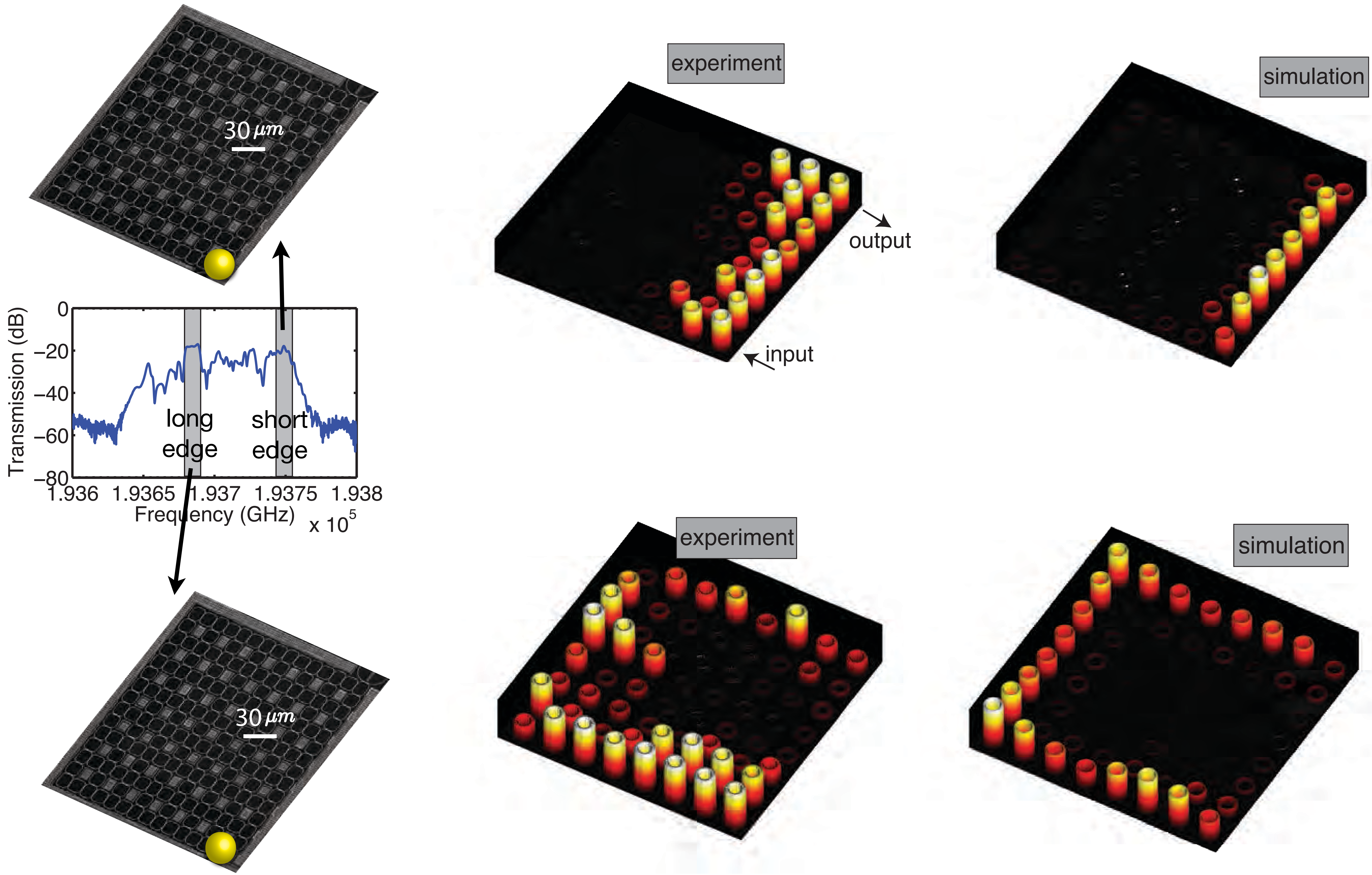
- Silicon-on-Insulator technology







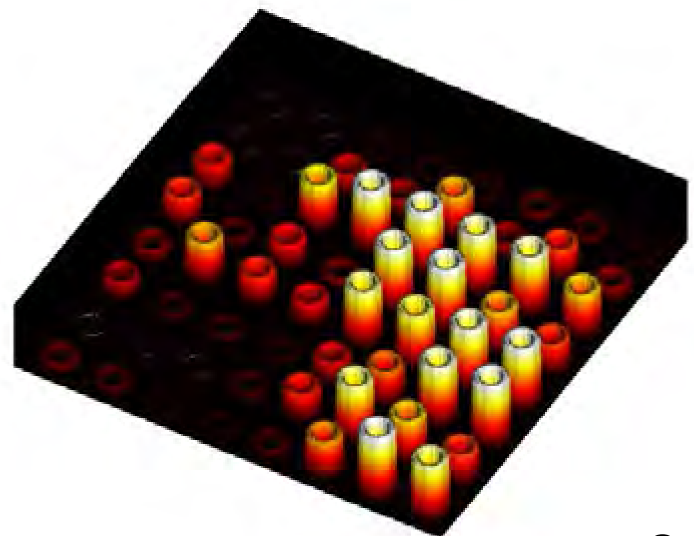
Observation of topological edge states



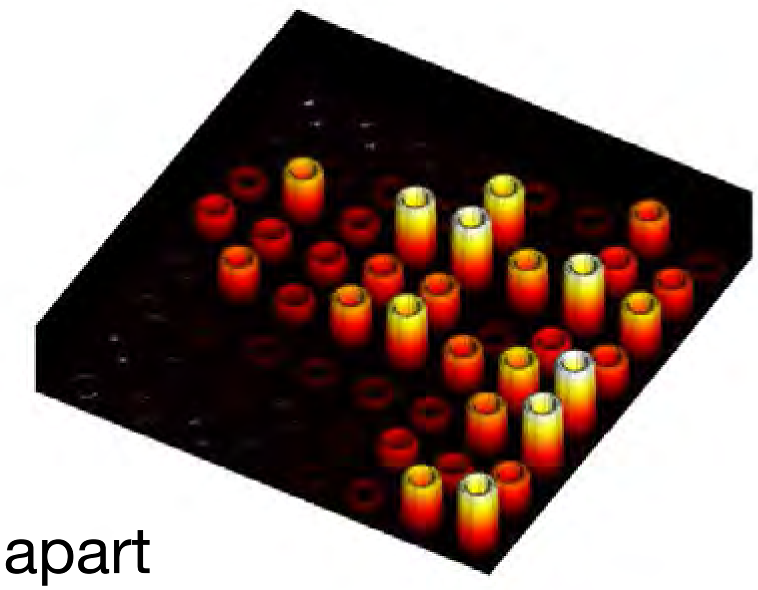
MH, Mittal, Fan, Migdall, Taylor
Nature Photon 7, 1001 (2013)

Topological robustness of edge state

Bulk state profile is extremely sensitive to frequency change.



0.1GHz apart

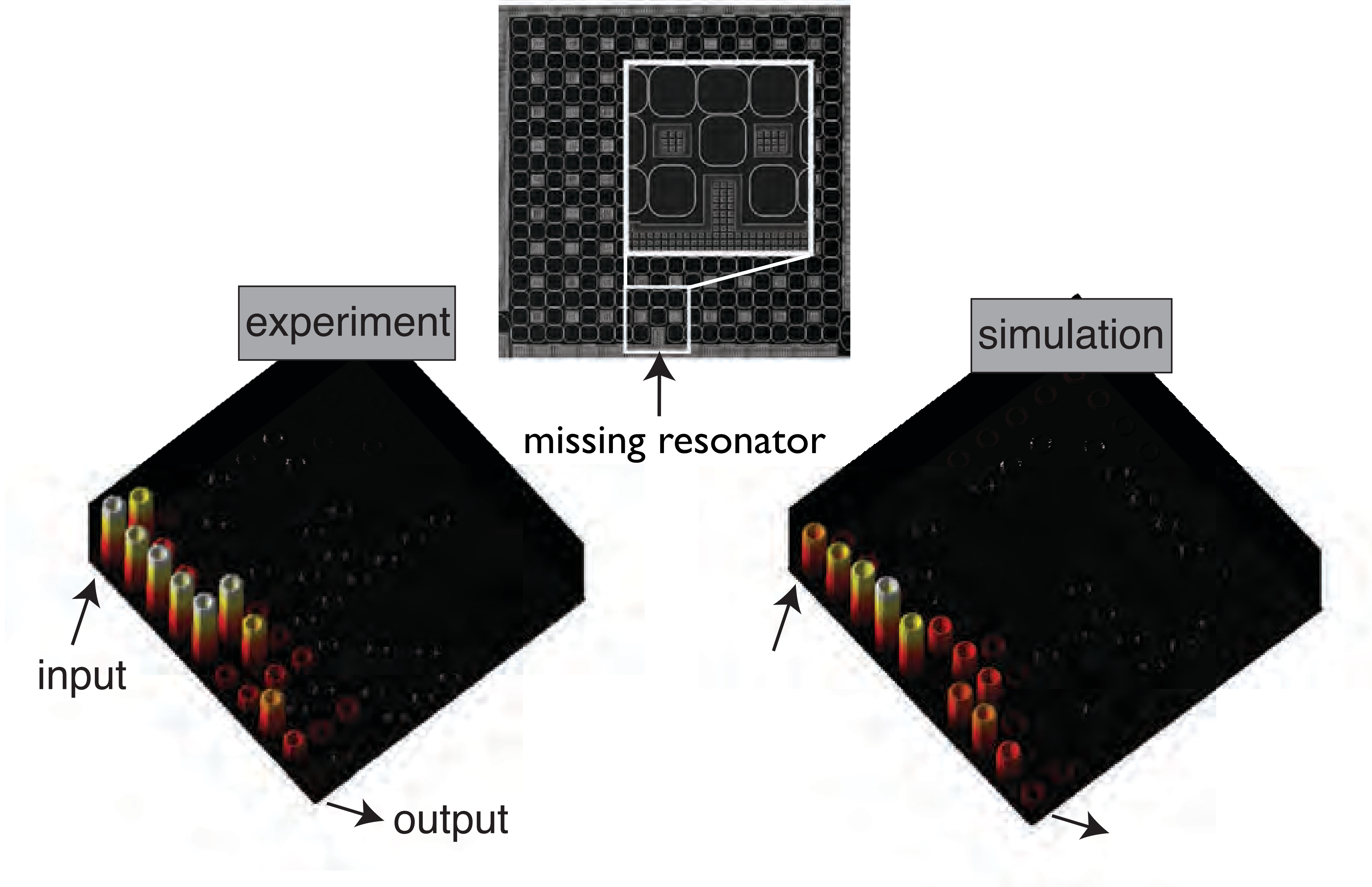


- Edge states are observed over a broad band (15 GHz)

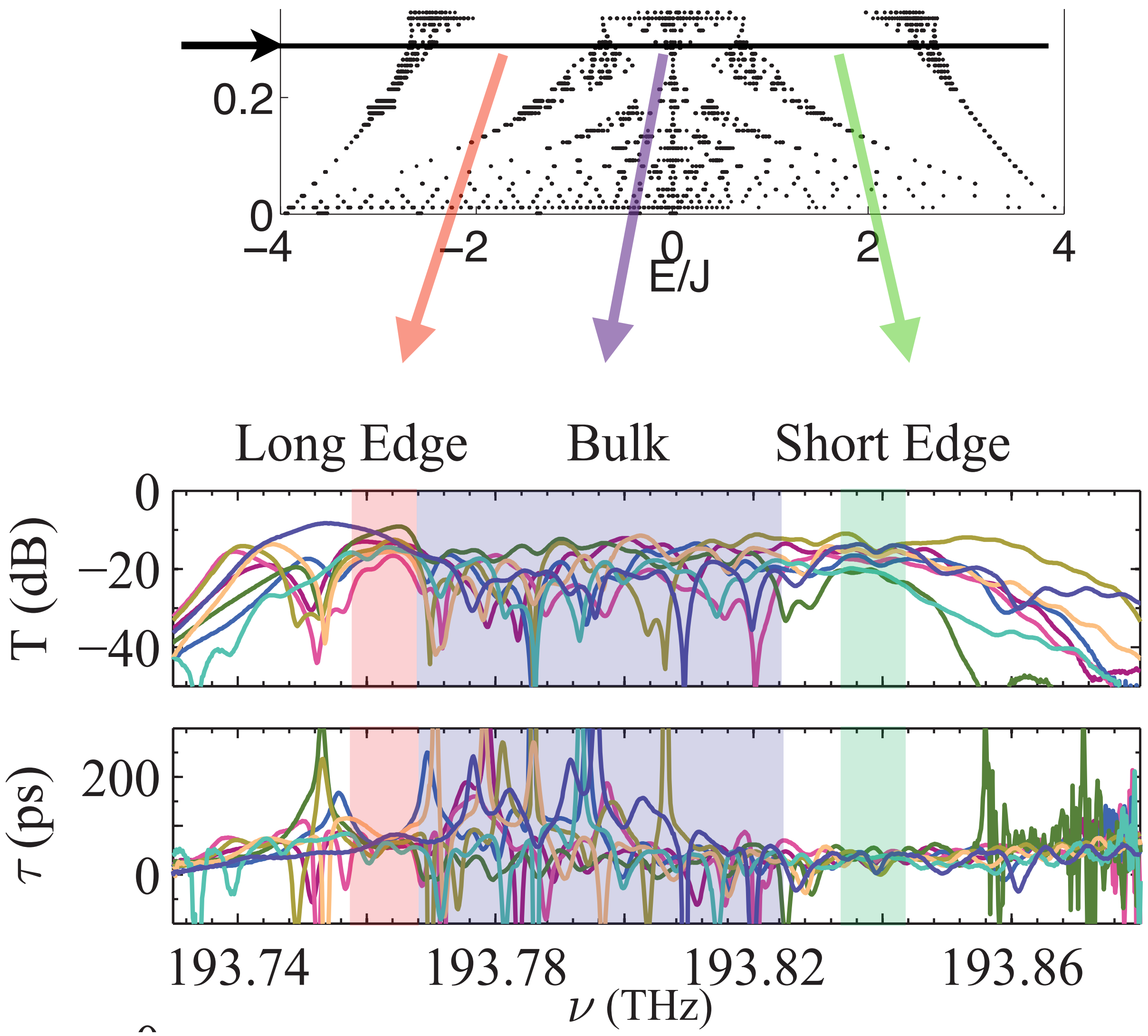
- Edge state is robust against intrinsic disorder and the propagation profile does not change

frequency mismatch	$U = 0.8J$
loss rate inhomogeneity	$\Delta\kappa_{in} = 0.45\kappa_{in}$
coupling inhomogeneity	$\Delta J = 0.04J$
magnetic field inhomogeneity	$\Delta\alpha = 2\pi 0.08$
spin flip rate	$\beta = 0.04J$

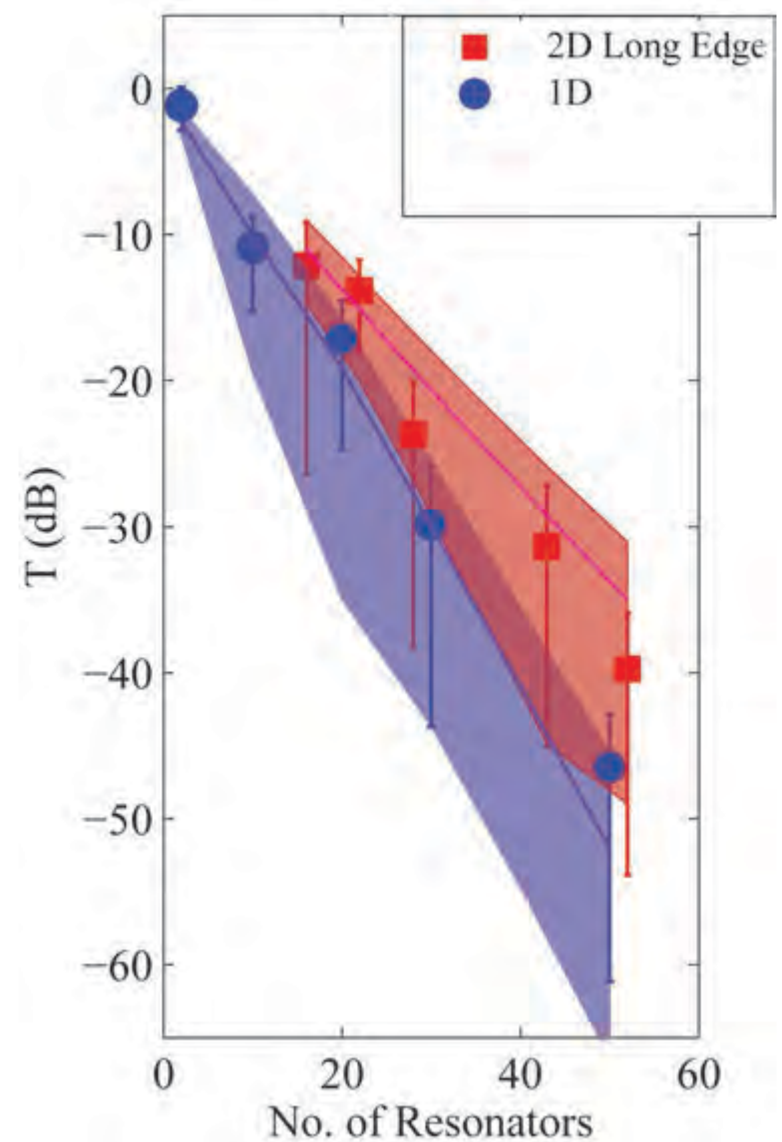
Robustness against an introduced disorder



Transport statistics



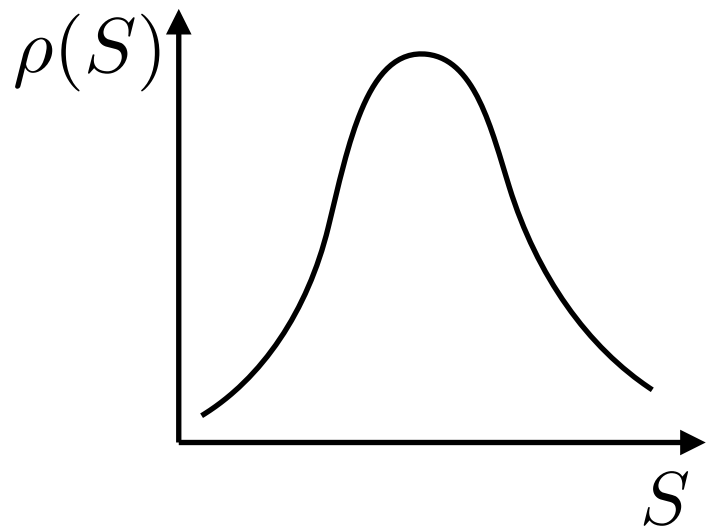
15x15 arrays
Different colors: different samples

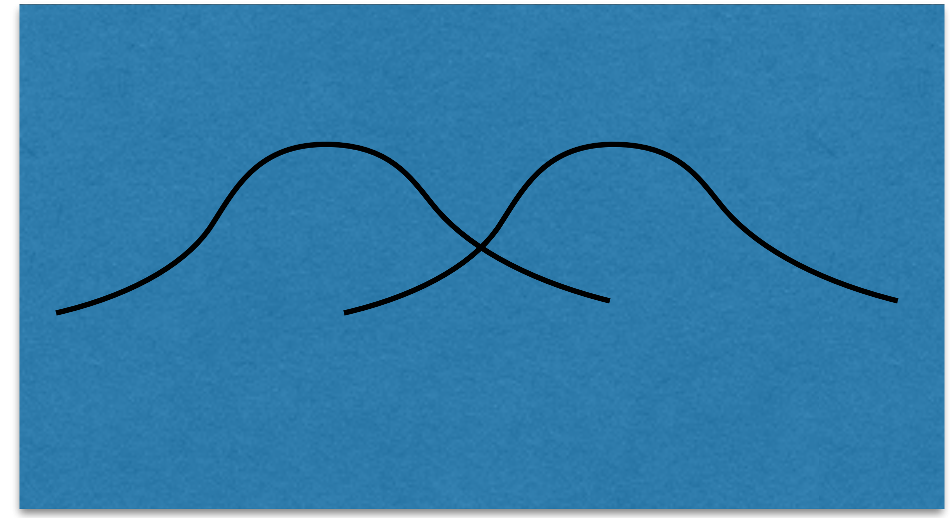


S. Mittal, J. Fan, S. Faez, A. Migdall, J. M. Taylor, and M.H. Hafezi,
Phys. Rev. Lett. 113, 087403 (2014)

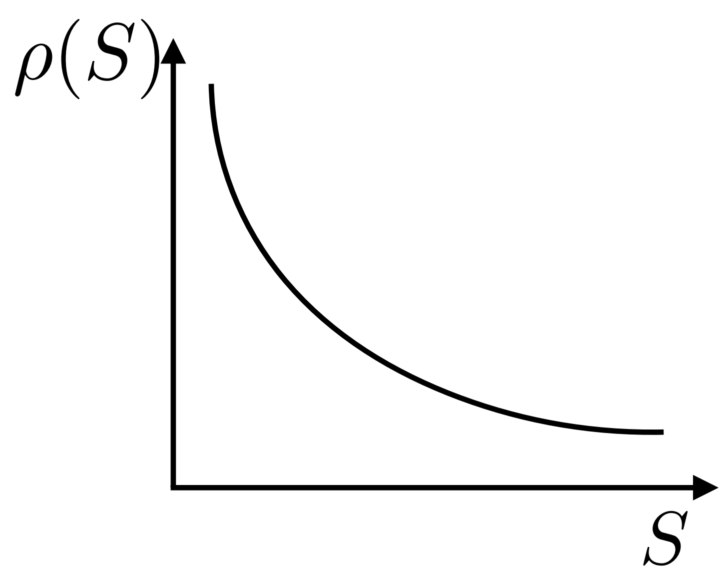
localization and level repulsion

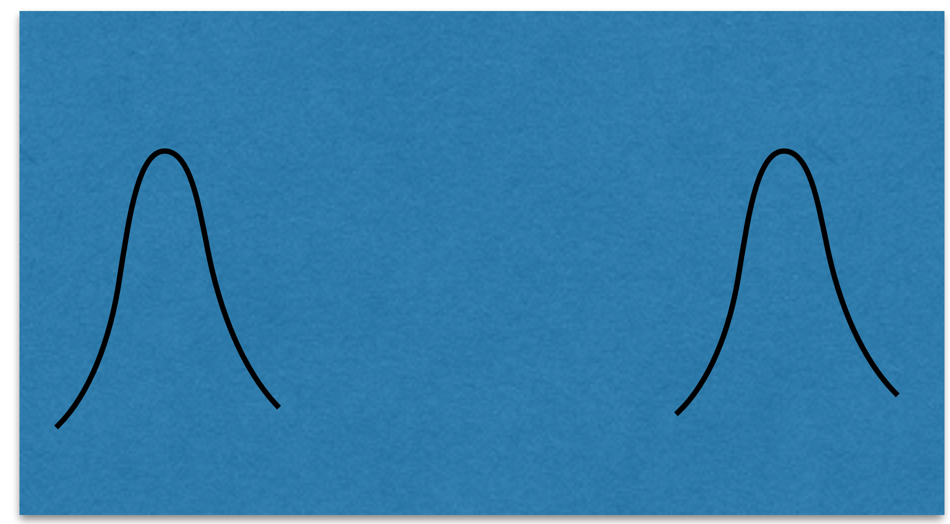
diffusive:



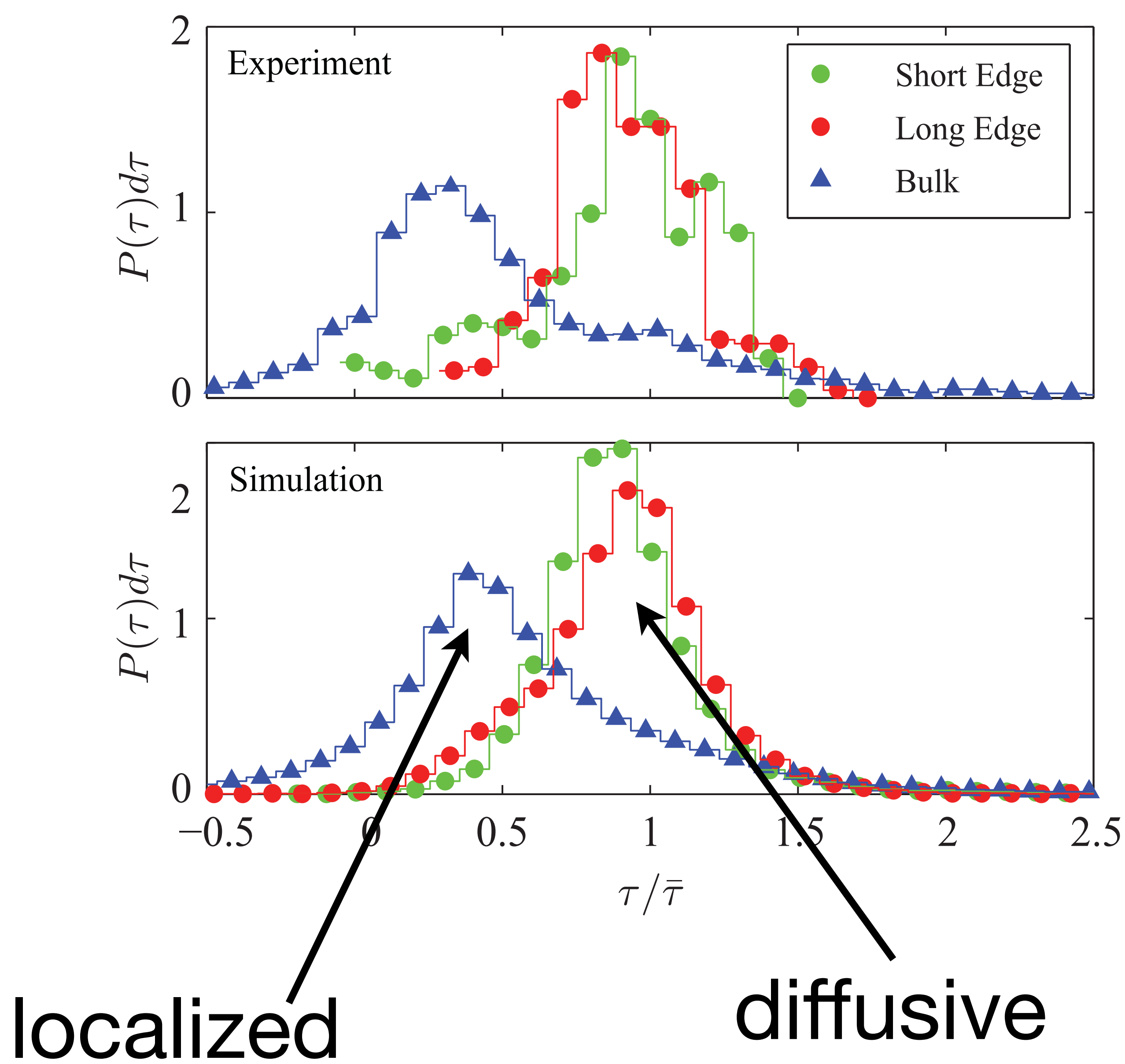


localized:





$$\tau(\omega) = \frac{L}{v_g(\omega)} \propto \frac{dk}{d\omega} \propto \rho(\omega)$$

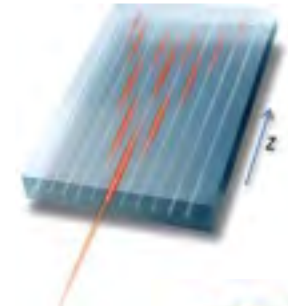


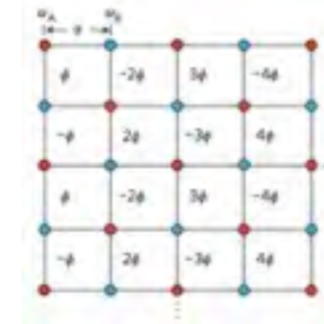
Asymmetricity and deviation of the peak from the mean indicates localization of bulk states

earlier work in 1D:
Genack Nature (2000)
Texier & Comtet (1999)

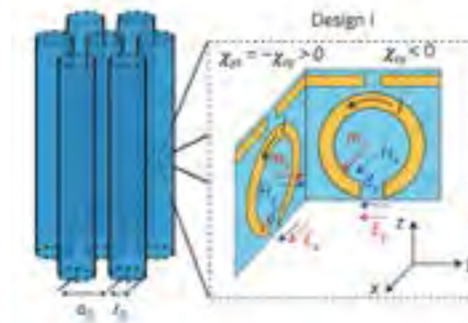
S. Mittal, J. Fan, S. Faez, A. Migdall, J. M. Taylor, and M. H.
Phys. Rev. Lett. 113, 087403 (2014)

Recent publications exploring topological properties of light

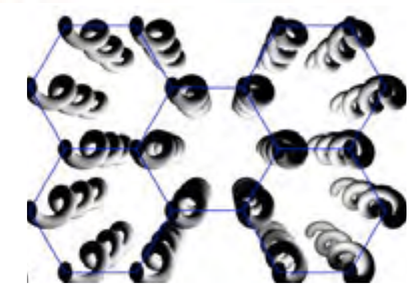




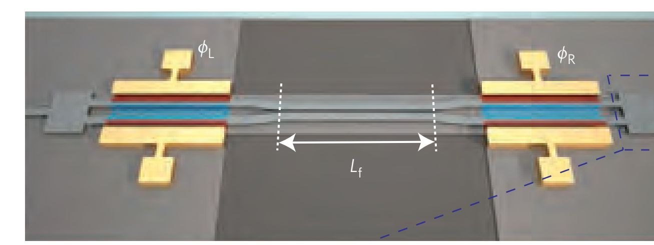
- YE Kraus, Y Lahini, Z Ringel, M Verbin, O Zilberberg - Physical Review Letters, 2012
- L. Lu, L. Fu, J. Joannopoulos and M. Soljacic Nature Photonics **7**, 294–299 (2013)
- V Yannopapas New Journal of Physics (2012)
- K Fang, Z Yu, S Fan - Nature Photonics (2012)
- M Rechtsman, et al. - Nature Photonics (2012)
- A. Khanikaev, S. Mousavi, W. Tse, M. Kargarian, A. MacDonald, G. Shvets, Nature Material (2012)



- W. Yang, Z. Yin, Z. Chen, S. Kou, M. Feng, C. Oh - Physical Review A (2012)



- M. Verbin, O. Zilberberg, Y. Kraus, Y. Lahini, and Y. Silberberg Physical Review Letter (2013)
- MC Rechtsman, et al. - Nature (2013)
- G. Jiang, Y. Chong Physical Review Letters (2013)
- V. G. Sala, D. D. Solnyshkov[3], T. Jacqmin[1], A. Lemaître[1], H. Terças[3], M. Abbarchi, E. Galopin, I. Sagnes, J. Bloch, G. Malpuech, A. Amo, Arxiv (2014)



- Jia Ningyuan, Ariel Sommer, David Schuster, Jonathan Simon Arxiv (2013)
- L. Tzuang … M. Lipson - Nature Photonics (2014)

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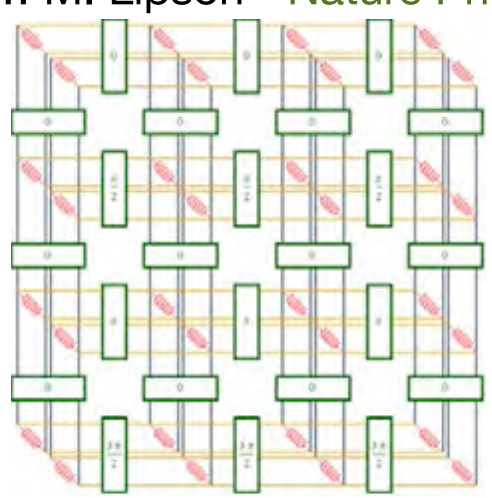

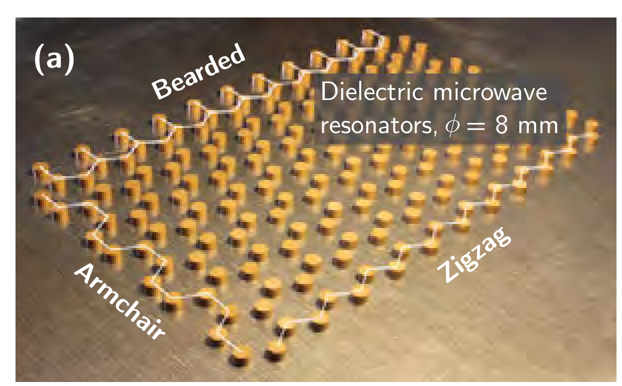

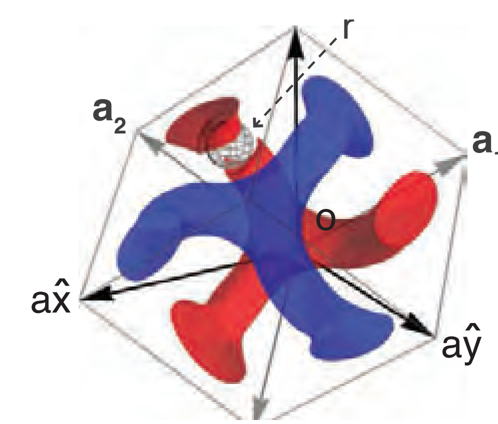

M. H. "Synthetic gauge fields with photons" Int. J. Mod. Phys. B 28, 1441002 (2014).